

Sequences

Problem 1. Write first few terms of the sequence, if the n -th term is determined by formula

a) $a_n = \frac{n+1}{2n}$

b) $a_n = \cos(n\pi)$,

c) $a_n = \frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n}{n^2}$.

Results:

a) $\{a_n\}_{n=1}^{\infty} = \left\{1, \frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{3}{5}, \frac{7}{12}, \frac{4}{7}, \frac{9}{16}, \frac{5}{9}, \frac{11}{20}, \dots\right\}$, b) $\{a_n\}_{n=1}^{\infty} = \{-1, 1, -1, 1, -1, 1, \dots\}$,

c) $\{a_n\}_{n=1}^{\infty} = \left\{1, \frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{3}{5}, \frac{7}{12}, \frac{4}{7}, \frac{9}{16}, \frac{5}{9}, \frac{11}{20}, \dots\right\}$.

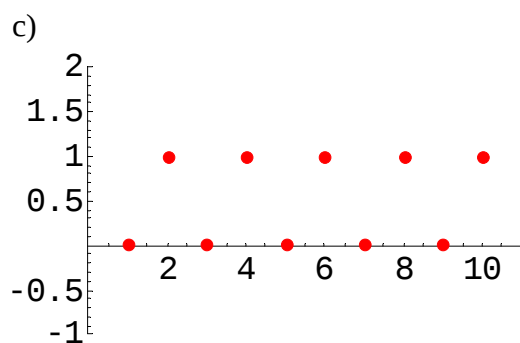
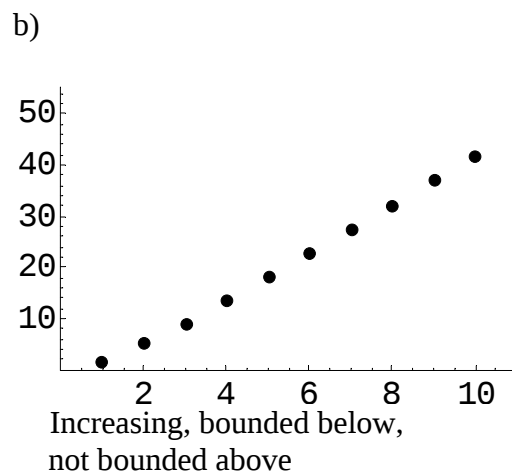
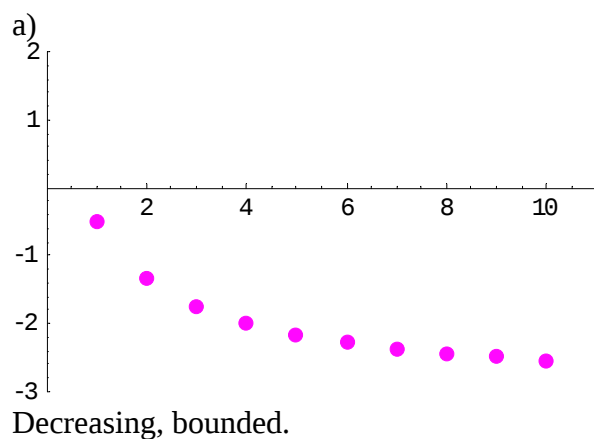
Problem 2. Deduce from the graph of the sequence whether the sequence is monotone and bounded, and prove your estimation.

a) $\left\{\frac{2-3n}{n+1}\right\}_{n=1}^{\infty}$

b) $\left\{\frac{5n^2}{n+2}\right\}_{n=1}^{\infty}$

c) $\left\{\frac{1+(-1)^n}{2}\right\}_{n=1}^{\infty}$

Results:



Limit of a sequence

Evaluate the following limits of sequences:

1. $\lim_{n \rightarrow \infty} \frac{3n+1}{2n-3}$	$\left(\frac{3}{2}\right)$
2. $\lim_{n \rightarrow \infty} \frac{3n^2+2n-5}{2+3n-n^2}$	(-3)
3. $\lim_{n \rightarrow \infty} \frac{-3n}{1+n-2n^2}$	(0)
4. $\lim_{n \rightarrow \infty} \left(\frac{n^5+1}{2n^5+3n}\right)^4$	$\left(\frac{1}{16}\right)$
5. $\lim_{n \rightarrow \infty} \sqrt{\frac{n^3-n+2}{4n^3+3n^2}}$	$\left(\frac{1}{2}\right)$
6. $\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^3+2n-1}}{n+2}$	(1)
7. $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2+3n}}{\sqrt[3]{2n^3-2n}}$	$\left(\frac{1}{\sqrt[3]{2}}\right)$
8. $\lim_{n \rightarrow \infty} 2^{\frac{2n^2-3n}{1-n^2}}$	$\left(\frac{1}{4}\right)$
9. $\lim_{n \rightarrow \infty} \left(2+\frac{1}{n}\right)^{\frac{2n^2-3n}{1-n^2}}$	$\left(\frac{1}{4}\right)$
10. $\lim_{n \rightarrow \infty} \left(3+\frac{1}{4^n}\right)$	(3)
11. $\lim_{n \rightarrow \infty} \frac{3^n+4}{1-2 \cdot 3^n}$	$\left(-\frac{1}{2}\right)$
12. $\lim_{n \rightarrow \infty} \left(\frac{1}{n^2}+\frac{2}{n^2}+\dots+\frac{n}{n^2}\right)$	$\left(\frac{1}{2}\right)$
13. $\lim_{n \rightarrow \infty} \sqrt[n]{2}$	(1)
14. $\lim_{n \rightarrow \infty} \sqrt[n]{4n}$	(1)
15. $\lim_{n \rightarrow \infty} \sqrt[n]{n^3}$	(1)
16. $\lim_{n \rightarrow \infty} \left(1+\frac{1}{n}\right)^{2n}$	(e^2)
17. $\lim_{n \rightarrow \infty} \left(1+\frac{1}{3n}\right)^n$	$(\sqrt[3]{e})$
18. $\lim_{n \rightarrow \infty} \left(1+\frac{1}{2n}\right)^{2n-1}$	(e)

19. $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+2} \right)^n$	(e)
20. $\lim_{n \rightarrow \infty} (n^2 - 2n + 1)$	(∞)
21. $\lim_{n \rightarrow \infty} (-3n^3 + 2n)$	$(-\infty)$
22. $\lim_{n \rightarrow \infty} \frac{n^3 - 2n}{n^2 + 3n + 1}$	(∞)
23. $\lim_{n \rightarrow \infty} \frac{n^3 - 2n}{1 - 2n^2}$	$(-\infty)$
24. $\lim_{n \rightarrow \infty} (\sqrt{n+4} - \sqrt{n+1})$	(0)
25. $\lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+2} \right)^{n+1}$	(∞)
26. $\lim_{n \rightarrow \infty} \left(\frac{2n+3}{3n+2} \right)^{2n}$	(0)
27. $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n} \right)^{n+1}$	(e^2)
28. $\lim_{n \rightarrow \infty} \left(\frac{n-1}{n+2} \right)^{\frac{n}{2}}$	$\left(\frac{1}{\sqrt{e^3}} \right)$