Number series

1. Fund sum of an infinite number series, if it exists:.

a) $\sum_{n=1}^{\infty}\frac{2}{n^2+4n+3}$, b) $\sum_{n=1}^{\infty}\frac{2}{n^2+11n+30}$, c) $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$.

d) $\sum_{n=1}^{\infty}\frac{2}{4^{n-1}}$, e) $\sum_{n=1}^{\infty}\left(\frac{-4}{5}\right)^n$, f) $\sum_{n=1}^{\infty}\frac{3^{n+1}}{2^n}$.

<u>*Solutions*</u>:

a) $\frac{5}{6}$, b) $\frac{1}{3}$, d) $\frac{8}{3}$, e) $-\frac{4}{9}$, c), f) does not exist.

2. Prove that the following infinite number series are divergent:

a) $\sum_{n=1}^{\infty}\frac{2n}{n+1}$, b) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[n]{n}}$, c) $\sum_{n=1}^{\infty}\cos\frac{1}{n}$, d) $\sum_{n=1}^{\infty}\sin\left(n\frac{\pi}{2}\right)$.

3. By means of the comparison test investigate convergence of the number series:

a) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}}$, b) $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n+1}}$, c) $\sum_{n=1}^{\infty}\frac{2}{n^2+3}$

<u>*Solutions*</u>:

a) divergent, b), c) convergent.

4. By means of the d'Alembert criterion investigate convergence of number series:

a) $\sum_{n=1}^{\infty}\frac{(n+1)^2}{3^n}$, b) $\sum_{n=1}^{\infty}\frac{5^n}{(n+1)!}$, c) $\sum_{n=1}^{\infty}\frac{(-1)^n n}{(n+1)!}$, d) $\sum_{n=1}^{\infty}\frac{2^n n!}{3^n}$,

e) $\sum_{n=1}^{\infty}\frac{1.3.5\ldots(2n-1)}{n!}$, f) $\sum_{n=1}^{\infty}\frac{n2^{n-1}}{3^n}$ g) $\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$ h) $\sum_{n=1}^{\infty}\frac{(n+1)!}{2^n.n!}$

<u>*Solutions*</u>:

a), b), c), f), g), h) convergent, d), e), divergent.

5. By means of the Cauchy criterion investigate convergence of number series:

a) $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^{2n}$, b) $\sum_{n=1}^{\infty}\left(\frac{5n}{3n-1}\right)^{\frac{2}{3}n}$, c) $\sum_{n=1}^{\infty}\left(\frac{n+2}{2n-1}\right)^n$,

d) $\sum_{n=1}^{\infty}\frac{n3^{n-1}}{2^n}$, e) $\sum_{n=1}^{\infty}\frac{(n+1)^2}{3^n}$ f) $\sum_{n=1}^{\infty}\operatorname{arctg}^n\frac{1}{n}$.

<u>*Solutions*</u>:

a), c), e) f) convergent, b), d), divergent.

6. By means of the Leibniz criterion investigate convergence of number series:

a) $\sum_{n=1}^{\infty}\frac{-1^{\;n}}{\sqrt{n}}$, b) $\sum_{n=1}^{\infty}\frac{-1^{\;n}}{n^3}$, c) $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n\ln n}$ d) $\sum_{n=2}^{\infty}\frac{-1^{\;n}}{n\ln^2 n}$

e) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$ f) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n.2^n}$ g) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+1)}{n}$ h) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\ln(n+1)}$

Is the respective series absolutely or relatively convergent?

Solutions:

a), c), e), h) relatively convergent, b), d), f) absolutely convergent, g) divergent.

7. Using the necessary condition for convergence of a number series prove that:

a) $\lim_{n\to\infty}\frac{e^n}{n!}=0$, b) $\lim_{n\to\infty}\operatorname{arctg}^n\frac{1}{n}=0$, c) $\lim_{n\to\infty}\frac{n!}{1.3.5\ldots\ 2n-1}=0$,

d) $\lim_{n\to\infty}\left(\frac{1}{3^n}+\frac{1}{5^n}\right)=0$ e) $\lim_{n\to\infty}\frac{1}{(2n-1)(2n+1)}=0$