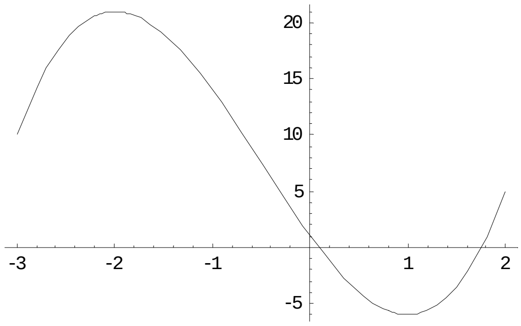
Maximal and minimal value of a function on an interval

1. Determine domain $D(f)$ of function $f: y = 2x^3 + 3x^2 - 12x + 1$. Find:
   a) all local extrema of function f on $D(f)$,
   b) global extrema (maximal and minimal value) of function f on interval $\langle -3, 2 \rangle$,
   c) whether function f is bounded on interval $\langle -3, 2 \rangle$.

Solution:
$D(f) = R$.

a) At point $x = -2$ is the local maximum with value $f(-2) = 21$, at point $x = 1$ is local minimum with value $f(1) = -6$.

b) $\max\limits_{x \in \langle -3,2 \rangle} f = f(-2) = 21$, $\min\limits_{x \in \langle -3,2 \rangle} f = f(1) = -6$.



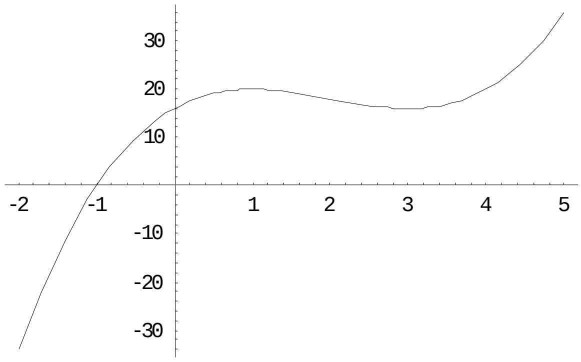
c) f is bounded on interval $\langle -3, 2 \rangle$

2. Determine domain $D(f)$ of function $f: y = x^3 - 6x^2 + 9x + 16$. Find:
   a) all local extrema of function f on $D(f)$,
   b) global extrema (maximal and minimal value) of function f on interval $\langle -2, 2 \rangle$ and range of function on interval $\langle -2, 2 \rangle$ (set of all points that are images of all points in interval $\langle -2, 2 \rangle$ in ther nmapping defined by function f).

Solution:
$D(f) = R$.

a) At the point $x = 3$ is local minimum with value $f(3) = 16$, at the point $x = 1$ is local maximum with value $f(1) = 20$,

b) $\max\limits_{x \in \langle -2,2 \rangle} f = f(1) = 20$, $\min\limits_{x \in \langle -2,2 \rangle} f = f(-2) = -34$, $f(\langle -2, 2 \rangle) = \langle -34, 20 \rangle$.



3. Determine domain $D(f)$ of function $f: y = \frac{1}{4}x^4 - x^3 + x^2$. Find:

   a) all local extrema of function f on $D(f)$,

   b) global extrema (maximal and minimal value) of function f on interval $\langle -1, 3 \rangle$,

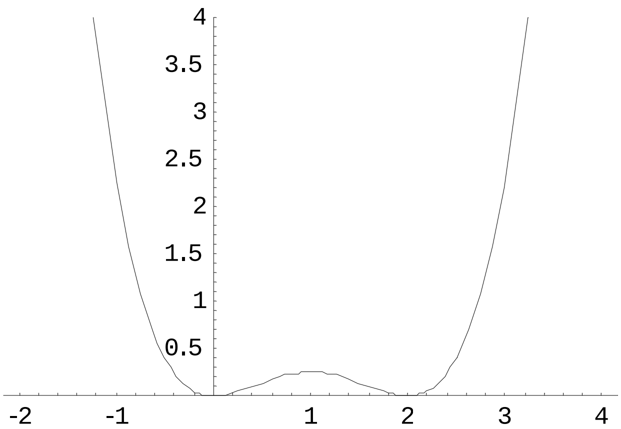
   c) whether function f is bounded below on $D(f)$.

***Solution**:*

$D(f) = R$.

a) At the point $x = 0$ is local minimum with value $f(0) = 0$, at the point $x = 1$ is local maximum with value $f(1) = \frac{1}{4}$, at the point $x = 2$ is local minimum with value $f(0) = 0$.

b) $\max\limits_{x \in \langle -1,3 \rangle} f = f(-1) = f(3) = \frac{9}{4}$, $\min\limits_{x \in \langle -1,3 \rangle} f = f(0) = f(2) = 0$.



c) f is bounded below