

Inverse function

Determine $D(f)$ and $H(f)$ and type of monotonicity of to the function $y = f(x)$ and find, whether there exists an inverse function. If such function exists, determine its formula.

1. $f : y = 2 - 2x$

$$D(f) = \mathbf{R}, H(f) = \mathbf{R}, \text{decreasing } f^{-1}(x) = \frac{2-x}{2}$$

2. $f(x) = x + 2\sqrt{x}$

$$D(f) = H(f) =]0, \infty), \text{increasing, } f^{-1}(x) = (\sqrt{x+1} - 1)^2$$

3. $f : y = 1 - \sqrt{2x-4}$

$$D(f) = \langle 2, \infty), H(f) = (-\infty, 1], \text{decreasing, } f^{-1}(x) = \frac{(x-1)^2}{2} + 2$$

4. $f : y = \frac{x+1}{x-2}$

$$D(f) = \mathbf{R} - \{2\}, H(f) = \mathbf{R} - \{1\}, \text{one-to-one, } f^{-1}(x) = \frac{2x+1}{x-1}$$

5. $f(x) = \frac{\sqrt{x}}{1+\sqrt{x}}$

$$D(f) =]0, \infty), H(f) =]0, 1], \text{increasing, } f^{-1}(x) = \left(\frac{x}{1-x}\right)^2$$

6. $f : y = 3 - \log_{\frac{1}{2}} x - 1$

$$D(f) = \langle 1, \infty), H(f) = \mathbf{R}, \text{increasing, } f^{-1}(x) = 1 - \left(\frac{1}{2}\right)^{x-3}$$

7. $f : y = 3e^{x+2}$

$$D(f) = \mathbf{R}, H(f) = (0, \infty), \text{increasing, } f^{-1}(x) = -2 + \ln \frac{x}{3}$$

8. $f(x) = \frac{1}{1+e^x}$

$$D(f) = \mathbf{R}, H(f) = (0, 1), \text{decreasing } f^{-1}(x) = \ln \frac{1-x}{x}$$

9. $f(x) = -\ln(10^x - 1)$

$$D(f) = (0, \infty), H(f) = \mathbf{R}, \text{decreasing, } f^{-1}(x) = \log(1 + e^{-x})$$

$$10. f : y = -2 \cot(3x + \pi)$$

$$D(f) = \left(-\frac{\pi}{3}, 0\right), H(f) = \mathbf{R}, \text{decreasing } f^{-1}(x) = \frac{-\operatorname{arccot}\left(-\frac{x}{2}\right) + \pi}{3}$$

$$11. f : y = 1 + 2 \arcsin 2 - x$$

$$D(f) = \langle 1, 3 \rangle, H(f) = \langle 1 - \pi, 1 + \pi \rangle, \text{decreasing } f^{-1}(x) = 2 - \sin \frac{x-1}{2}$$

$$12. f : 3 \operatorname{arccot} 2x + 1$$

$$D(f) = \mathbf{R}, H(f) = (0, 3\pi), \text{decreasing } f^{-1}(x) = \frac{-1 + \cot g \frac{x}{3}}{2}$$

$$13. f : y = -2 + \operatorname{arctg} 2x$$

$$D(f) = \mathbf{R}, H(f) = \left(-2 - \frac{\pi}{2}, -2 + \frac{\pi}{2}\right), \text{increasing } f^{-1}(x) = \frac{\operatorname{tg}(x+3)}{2}$$

$$14. f : y = 1 - \arccos \frac{x+1}{3}$$

$$D(f) = \langle -4, 2 \rangle, H(f) = (1 - \pi, 1), \text{increasing } f^{-1}(x) = 3 \cos(1 - x) - 1$$

$$15. f(x) = \sin(\arctan x)$$

$$D(f) = \mathbf{R}, H(f) = (-1, 1), \text{increasing } f^{-1}(x) = \tan(\arcsin x)$$

$$16. f(x) = \cos(\operatorname{arccot} x)$$

$$D(f) = \mathbf{R}, H(f) = (-1, 1), \text{increasing } f^{-1}(x) = \cot(\arccos x)$$