Gauss Elimination Method

Problem 1: Solve the system of linear equations:

a) $\begin{array}{rcrcrcr} x & & & + & z & = & 14 \\ 2x & + & 3y & + & z & = & 21 \\ -2x & + & 2y & + & 2z & = & -14 \end{array}$, b) $\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 1 \\ 2x & + & 3y & + & z & = & 2 \\ x & + & 3y & - & 2z & = & 1 \end{array}$,

c) $\begin{array}{rcrcrcr} x & + & y & + & 2z & = & 0 \\ x & - & 2y & + & z & = & 0 \\ -x & + & 2y & + & 2z & = & 0 \end{array}$.

Results:

a) $(x,y,z)=(10,-1,4)$, b) $(x,y,z)=(1,0,0)$, c) $(x,y,z)=(0,0,0)$.

Problem 2: Solve the system of linear equations:

a) $\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 1 \\ 2x & + & 3y & + & z & = & 2 \\ x & + & y & + & 2z & = & 1 \end{array}$, b) $\begin{array}{rcrcrcr} x & + & y & + & 2z & = & 4 \\ x & - & 2y & + & z & = & 0 \\ x & - & 5y & & & = & -4 \end{array}$,

c) $\begin{array}{rcrcrcr} 2x & - & y & + & z & = & 0 \\ x & + & 2y & - & 2z & = & 0 \\ 3x & + & y & - & z & = & 0 \end{array}$.

Results:

a) $(x,y,z)=(1-5t,3t,t),\ t\in R$, b) $(x,y,z)=\left(\frac{8-5t}{3},\frac{4-t}{3},t\right),\ t\in R$,

c) $(x,y,z)=(0,t,t),\ t\in R$.

Problem 3: Solve the system of linear equations:

a) $\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 1 \\ 2x & + & 3y & + & z & = & 2 \\ x & + & y & + & 2z & = & -1 \end{array}$, b) $\begin{array}{rcrcrcr} x & + & y & + & 2z & = & 4 \\ x & - & 2y & + & z & = & 0 \\ x & - & 5y & & & = & 2 \end{array}$.

Results:

a) System has no solution. b) System has no solution.

Problem 4: Solve the system of linear equations:

a) $$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & 3x_3 & + & x_4 & = & 5 \\ 2x_1 & - & x_2 & & & - & x_4 & = & -2 \\ 3x_1 & - & x_2 & + & x_3 & - & 2x_4 & = & -3 \\ x_1 & + & x_2 & & & + & x_4 & = & 3 \end{array}$$

b) $$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & & & - & 3x_4 & = & 0 \\ 7x_1 & - & 2x_2 & + & 2x_3 & - & 10x_4 & = & 0 \\ 7x_1 & - & x_2 & + & x_3 & - & 9x_4 & = & 0 \\ 2x_1 & & & - & 2x_3 & - & 4x_4 & = & 0 \\ 6x_1 & - & x_2 & + & 2x_3 & - & 7x_4 & = & 0 \end{array}$$

c) $$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & & & - & 3x_4 & = & 0 \\ 7x_1 & - & 2x_2 & + & 2x_3 & - & 10x_4 & = & 0 \\ 7x_1 & - & x_2 & + & x_3 & - & 9x_4 & = & 0 \\ 2x_1 & & & - & 2x_3 & - & 4x_4 & = & 0 \\ 6x_1 & - & x_2 & + & 2x_3 & - & 7x_4 & = & 1 \end{array}$$

d) $$\begin{array}{rcrcrcrcrcr} x_1 & + & x_2 & + & x_3 & + & x_4 & & & = & 1 \\ 2x_1 & + & 2x_2 & + & 2x_3 & & & & & = & 0 \\ x_1 & + & x_2 & + & 5x_3 & - & x_4 & + & 6x_5 & = & 1 \\ x_1 & + & x_2 & - & 3x_3 & + & x_4 & - & 6x_5 & = & -1 \end{array}$$

Results:

a) $\left(x_1, x_2, x_3, x_4\right) = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}, 2\right),\ t \in R,$

b) $\left(x_1, x_2, x_3, x_4\right) = \left(8t, -13t, -6t, 7t\right),\ t \in R,$

c) system has no solution,

d) $\left(x_1, x_2, x_3, x_4, x_5\right) = \left(\frac{-1+3t}{2} - p, p, \frac{1-3t}{2}, 1, t\right),\ t, p \in R$

Problem 5: Find all solutions of the system: $$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & 3x_3 & + & x_4 & = & 5 \\ 2x_1 & - & x_2 & & & - & x_4 & = & -2 \\ 3x_1 & - & x_2 & + & x_3 & - & 2x_4 & = & -3 \\ x_1 & & & + & x_3 & + & x_4 & = & 3 \end{array}$$

and then identify such solution $\left(x_1, x_2, x_3, x_4\right)$, for which the following holds $x_1 = x_4$.

Results:

$\left(x_1, x_2, x_3, x_4\right) = \left(1-t, 2-2t, t, 2\right),\ t \in R$

$x_1 = x_4 \Leftrightarrow \left(x_1, x_2, x_3, x_4\right) = \left(2, 4, -1, 2\right).$

Problem 6: Find all solutions of the system:

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & - & x_4 & = & 2 \\ x_1 & + & 3x_2 & - & x_3 & + & x_4 & = & -8 \\ 2x_1 & + & x_2 & - & 3x_3 & & & = & -6 \\ -2x_1 & + & 2x_2 & + & 2x_3 & + & 2x_4 & = & -4 \end{array}$$

and then identify such solution (x_1, x_2, x_3, x_4), for which the following holds $x_1 + x_4 = 0$.

Results:

$(x_1, x_2, x_3, x_4) = \left(2t+2, -\frac{5}{2}-t, \frac{5}{2}+t, 2t\right),\ t \in R$

$x_1 + x_4 = 0 \Leftrightarrow (x_1, x_2, x_3, x_4) = (1, -2, 2, -1)$.