

Determinants

Problem 1: Evaluate determinants:

a) $\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix},$

b) $\begin{vmatrix} 3 & 2 \\ 1,5 & 1 \end{vmatrix},$

c) $\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix},$

d) $\begin{vmatrix} e^x & e^{-2x} \\ e^x & -2e^{-2x} \end{vmatrix}$

Results:

a) 3, b) 0, c) 1, d) $-3e^{-x}$.

Problem 2: Evaluate determinants:

a) $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ -2 & 2 & 2 \end{vmatrix},$

b) $\begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & 2 \\ -1 & 2 & 2 \end{vmatrix},$

c) $\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 3 & 1 & 0 \end{vmatrix}.$

Results:

a) 14, b) 7, c) 0.

Problem 3: Using Crammer rule solve the system of 2 equations with 2 unknowns:

a)
$$\begin{array}{rcl} x & + & 2y = 3 \\ -x & + & y = 9' \end{array}$$

b)
$$\begin{array}{rcl} 3x & - & 2y = 4 \\ \frac{3}{2}x & - & y = 2' \end{array}$$

Results:

a) $(x,y)=(-5,4),$ b) $D=0$, cannot be solved using Crammer rule.

Problem 4: By means of Crammer rule solve the system of 3 equations with 3 unknowns:

a)
$$\begin{array}{rcl} x & + & z = 14 \\ 2x & + & 3y + z = 21, \\ -2x & + & 2y + 2z = -14 \end{array}$$

b)
$$\begin{array}{rcl} -y & + & z = 3 \\ x & + & y + 2z = 1, \\ -x & + & 2y + 2z = -1 \end{array}$$

c)
$$\begin{array}{rcl} x & + & y + z = 2 \\ -x & + & y + 2z = 0. \\ 3x & + & y = 4 \end{array}$$

Results:

a) $(x,y,z)=(10,-1,4),$ b) $(x,y,z)=\left(\frac{1}{7}, -\frac{12}{7}, \frac{9}{7}\right),$
c) $D=0$, cannot be solved by means of Crammer rule.