

Derivatives

Problem 1. Find derivative of function f :

a) $f(x) = 2x^3 - 4x^2 + x - 2$

$$(f'(x) = 6x^2 - 8x + 1),$$

b) $f(x) = x + 4\sqrt{x} - \sqrt[3]{x^2} + 2\frac{1}{\sqrt{x}}$

$$\left(f'(x) = 1 + \frac{2}{\sqrt{x}} - \frac{2}{3\sqrt[3]{x}} - \frac{1}{\sqrt{x^3}}\right),$$

c) $f(x) = 2x - \frac{4}{x} + \frac{1}{x^3}$

$$\left(f'(x) = 2 + \frac{4}{x^2} - \frac{3}{x^4}\right),$$

d) $f(x) = \frac{2x}{x^2 + 1}$

$$\left(f'(x) = \frac{2 - 2x^2}{(x^2 + 1)^2}\right),$$

e) $f(x) = (\sqrt{x} - 1) \cdot (\sqrt{x} + 3)$

$$\left(f'(x) = \frac{\sqrt{x} + 1}{\sqrt{x}}\right)$$

f) $f(x) = (e^x + 1) \cdot (x^2 - 2x)$

$$(f'(x) = x^2 \cdot e^x - 2e^x + 2x - 2),$$

g) $f(x) = \frac{\ln x + 1}{x + 1}$

$$\left(f'(x) = \frac{1 - x \ln x}{x(x + 1)^2}\right),$$

h) $f(x) = \sin x \cdot (2^x + \log_2 x)$

$$\left(f'(x) = \cos x \cdot (2^x + \log_2 x) + \sin x \cdot \left(2^x \ln 2 + \frac{1}{x \ln 2}\right)\right),$$

i) $f(x) = \frac{1 + \sin x}{1 + \cos x}$

$$\left(f'(x) = \frac{1 + \cos x + \sin x}{(1 + \cos x)^2}\right),$$

j) $f(x) = \operatorname{tg} x - \operatorname{cot} g x$

$$\left(f'(x) = \frac{1}{\sin^2 x \cdot \cos^2 x}\right),$$

k) $f(x) = x \cdot \operatorname{arctg} x$

$$\left(f'(x) = \operatorname{arctg} x + \frac{x}{1 + x^2}\right),$$

l) $f(x) = \frac{x^2 + 1}{\operatorname{arc} \cot g x}$

$$\left(f'(x) = \frac{2x \cdot \operatorname{arc} \cot g x + 1}{\operatorname{arc} \cot g^2 x}\right),$$

m) $f(x) = 2 \arcsin x - \arccos x$

$$\left(f'(x) = \frac{3}{\sqrt{1 - x^2}}\right).$$

Problem 2. Determine derivative of function f :

$$\text{a) } f(x) = (x^2 + 4x + 2)^9 \quad \left(f'(x) = 9(x^2 + 4x + 2)^8 \cdot (2x + 4) \right)$$

$$\text{b) } f(x) = \sqrt{\frac{1-x}{1+x}} \quad \left(f'(x) = -\frac{1}{(x+1)\sqrt{1-x^2}} \right)$$

$$\text{c) } f(x) = \sqrt[3]{5x+4} \quad \left(f'(x) = \frac{5}{3\sqrt[3]{(5x+4)^2}} \right)$$

$$\text{d) } f(x) = \frac{1}{x^2 + 1} \quad \left(f'(x) = \frac{-2x}{(x^2 + 1)^2} \right)$$

Problem 3. Find derivative of function f :

$$\text{a) } f(x) = 2^{\sqrt{x}} \quad \left(f'(x) = \frac{2^{\sqrt{x}} \ln 2}{2\sqrt{x}} \right)$$

$$\text{b) } f(x) = e^{-x^2+3} \quad \left(f'(x) = -2xe^{-x^2+3} \right)$$

$$\text{c) } f(x) = \log_2(\operatorname{tg} x) \quad \left(f'(x) = \frac{1}{\operatorname{tg} x \cdot \ln 2 \cdot \cos^2 x} \right)$$

$$\text{d) } f(x) = \ln(x + \sqrt{1+x^2}) + \sqrt{2} \quad \left(f'(x) = \frac{1}{\sqrt{1+x^2}} \right)$$

$$\text{e) } f(x) = \ln x^2 - \ln^2 x \quad \left(f'(x) = \frac{2-2\ln x}{x} \right)$$

Problem 4. Find derivative of function f :

$$\text{a) } f(x) = \sin 3x + \sin^3 x + \sin x^3 \quad \left(f'(x) = 3\cos 3x + 3\sin^2 x \cos x + 3x^2 \cos x^3 \right)$$

$$\text{b) } f(x) = e^{2x} \cos 3x \quad \left(f'(x) = e^{2x} (2\cos 3x - 3\sin 3x) \right)$$

$$\text{c) } f(x) = \sqrt{\pi^2 + \cot g x} \quad \left(f'(x) = -\frac{1}{2\sqrt{\pi^2 + \cot g x} \cdot \sin^2 x} \right)$$

Problem 5. Find derivative of function f :

$$\text{a) } f(x) = \arcsin \sqrt{x} \quad \left(f'(x) = \frac{1}{2\sqrt{x-x^2}} \right)$$

$$\text{b) } f(x) = \frac{1}{\arccos^2 x} \quad \left(f'(x) = \frac{2}{\arccos x \cdot \sqrt{1-x^2}} \right)$$

$$\text{c) } f(x) = \operatorname{arctg} \frac{x+1}{x-1} \quad \left(f'(x) = -\frac{1}{1+x^2} \right)$$

$$\text{d) } f(x) = \sqrt[3]{\operatorname{arc} \cot g^4 x} \quad \left(f'(x) = -\frac{4\sqrt[3]{\operatorname{arc} \cot g^4 x}}{3(1+x^2)} \right)$$

Problem 6. Find derivative of function f :

- a) $f(x) = x^x$ $\left(f'(x) = x^x (\ln x + 1) \right)$
- b) $f(x) = (\sin x)^x$ $\left(f'(x) = (\sin x)^x (\ln(\sin x) + x \cdot \cot x) \right)$
- c) $f(x) = (1+x^2)^{\arctg x}$ $\left(f'(x) = (1+x^2)^{\arctg x} \cdot \frac{\ln(1+x^2) + 2x \cdot \arctg x}{1+x^2} \right)$
- d) $f(x) = x^{\tg x}$ $\left(f'(x) = x^{\tg x} \cdot \left(\frac{\ln x}{\cos^2 x} + \frac{\tg x}{x} \right) \right)$

Problem 7. Evaluate:

- a) $f''(x)$, ak $f(x) = \frac{1}{x}$ $\left(f''(x) = \frac{2}{x^3} \right)$
- b) $f^{(4)}(x)$, ak $f(x) = 7x^3 - 4x^2 + 5$ $\left(f^{(4)} = 0 \right)$
- c) $f'''(x)$, ak $f(x) = \sqrt{x}$ $\left(f'''(x) = \frac{3}{8x^2\sqrt{x}} \right)$
- d) $f''(x)$, ak $f(x) = \arctg x$ $\left(f''(x) = -\frac{2x}{(1+x^2)^2} \right)$

Problem 8. Determine equation of a tangent line to the graph of function f at the point T .

- a) $f(x) = \frac{2x+1}{x^2}$ $T = [-2, ?]$ $(x + 4y - 1 = 0)$
- b) $f(x) = xe^{\frac{1}{x}}$ $T = [2, ?]$ $(\sqrt{e}x - 2y + 2\sqrt{e} = 0)$
- c) $f(x) = \ln \sin x$ $T = \left[\frac{\pi}{2}, ? \right]$ $(y = 0)$