

Definite and improper integrals

Evaluate:

1. $\int_0^2 (x^2 - 3x + 2) dx$ $\left(\frac{2}{3}\right)$
2. $\int_0^3 (\sqrt[3]{x} + \sqrt{3x}) dx$ $\left(6 + \frac{9\sqrt{3}}{4}\right)$
3. $\int_{-2}^1 |x+1| dx$ $\left(\frac{5}{2}\right)$
4. $\int_0^1 \frac{1}{1+x^2} dx$ $\left(\frac{\pi}{4}\right)$
5. $\int_3^7 \frac{x}{x^2-4} dx$ $(\ln 3)$
6. $\int_0^{\frac{\pi}{3}} \operatorname{tg} x dx$ $(\ln 2)$
7. $\int_0^{2\pi} \sqrt{1-\sin^2 x} dx$ (4)

Per partes method:

1. $\int_0^1 e^x (1-x) dx$ $(e-2)$
2. $\int_0^{\pi} (2x+1) \sin x dx$ $(2\pi+2)$
3. $\int_0^{\sqrt{3}} x \operatorname{arctg} x dx$ $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$
4. $\int_1^e x \ln x dx$ $\left(\frac{e^2+1}{4}\right)$
5. $\int_0^1 \operatorname{arctg} x dx$ $\frac{\pi}{4} - \frac{\ln 4}{4}$
6. $\int_1^e \ln^2 x dx$ $(e-2)$

Substitution method:

$$1. \int_3^5 \sqrt{x-3} \, dx \quad \left(\frac{4\sqrt{2}}{3} \right)$$

$$2. \int_0^1 \frac{x}{\sqrt{1+x^2}} \, dx \quad (\sqrt{2}-1)$$

$$3. \int_1^e \frac{1+\ln x}{x} \, dx \quad \left(\frac{3}{2} \right)$$

$$4. \int_0^1 \frac{\arctg x}{1+x^2} \, dx \quad \frac{\pi^2}{32}$$

$$5. \int_0^{\frac{\pi}{4}} \frac{\sqrt{\tg x + 1}}{\cos^2 x} \, dx \quad \frac{2}{3}(2\sqrt{2}-1)$$

$$6. \int_0^{\pi} \sin^2 x \, dx \quad \left(\frac{\pi}{2} \right)$$

$$7. \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 x \, dx \quad \left(\frac{4}{3} \right)$$

$$8. \int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}} \, dx \quad (\ln 4 - 1)$$

Investigate convergence of improper integrals:

$$1. \int_3^{\infty} \frac{1}{\sqrt{(x-2)^3}} \, dx \quad (2)$$

$$2. \int_0^{\infty} \frac{3x}{x^2+4} \, dx \quad (\infty; \text{divergentný})$$

$$3. \int_2^{\infty} \frac{\ln x}{x} \, dx \quad (\infty; \text{divergentný})$$

$$4. \int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} \, dx \quad (e-1)$$

$$5. \int_{-\infty}^0 e^{2x} \, dx \quad \left(\frac{1}{2} \right)$$

$$6. \int_{-\infty}^1 \frac{1}{1+x^2} \, dx \quad \left(\frac{3}{4}\pi \right)$$

$$7. \int_2^3 \frac{1}{\sqrt{x-2}} dx$$

$$(2)$$

$$8. \int_{-1}^0 \frac{x+2}{\sqrt{x+1}} dx$$

$$\left(\frac{8}{3}\right)$$

$$9. \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos^2 x} dx$$

$$(\infty; \textit{divergentný})$$

$$10. \int_{-\frac{\pi}{2}}^0 tgx dx$$

$$(-\infty; \textit{divergentný})$$

$$11. \int_{-2}^2 \frac{1}{\sqrt{4-x^2}} dx$$

$$\pi$$