Continuity of a function

<u>Problem 1.</u> Find points of discontinuity of function and evaluate one-sided limits or limits a these points.

a) $f: y=\dfrac{1}{x^2-1}$, b) $f: y=\dfrac{x+1}{x^2-1}$, c) $f: y=\dfrac{x^2-1}{x-1}$, d) $f: y=\dfrac{x+2}{x+2}$.

Results:

a) $1,\ -1,\ \lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to -1^-} f(x)=\infty,\ \lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to -1^+} f(x)=-\infty$,

b) $1,\ -1,\ \lim\limits_{x\to 1^+} f(x)=\infty,\ \lim\limits_{x\to 1^-} f(x)=-\infty,\ \lim\limits_{x\to -1} f(x)=-\dfrac{1}{2}$,

c) $1,\ \lim\limits_{x\to 1} f(x)=2$,

d) $-2,\ \lim\limits_{x\to -2} f(x)=1$.

<u>Príklad 2.</u> Find out the image of the interval J in the mapping defined by function *f*, if:

a) $f: y=x^2+3,\ J=\langle 1,5\rangle$, b) $f: y=\dfrac{1}{x+2},\ J=\langle -1,1\rangle$,

c) $f: y=-\ln x,\ J=\langle 1,e^2\rangle$, d) $f: y=1+\arccos\dfrac{x}{2},\ J=\langle 0,1\rangle$,

e) $f: y=2\cos 2x,\ J=\left\langle 0,\dfrac{\pi}{2}\right\rangle$, f) $f: y=\operatorname{arctg} x,\ J=\langle -1,1\rangle$.

Results:

a) $\langle 4,28\rangle$, b) $\left\langle \dfrac{1}{3},1\right\rangle$, c) $\langle -2,0\rangle$, d) $\left\langle 1+\dfrac{\pi}{3},1+\dfrac{\pi}{2}\right\rangle$, e) $\langle -2,2\rangle$, f) $\left\langle -\dfrac{\pi}{4},\dfrac{\pi}{4}\right\rangle$.

<u>Problem 3.</u> Prove that equation ($f(x)=0$) has at least one root on interval J:

a) $x^3-x-1=0,\ J=\langle 0,2\rangle$, b) $\cos x-x=0,\ J=\left\langle 0,\dfrac{\pi}{2}\right\rangle$,

c) $e^x+x=0,\ J=\langle -1,0\rangle$, d) $\operatorname{arctg} x-x^2=0,\ J=\left\langle \dfrac{1}{\sqrt{3}},\sqrt{3}\right\rangle$.

Results:

a) $f(1)=-1,\ f(2)=5$, b) $f(0)=1,\ f\left(\dfrac{\pi}{2}\right)=-\dfrac{\pi}{2}$, c) $f(-1)=\dfrac{1}{e}-1,\ f(0)=1$,

d) $f\left(\dfrac{1}{\sqrt{3}}\right)=\dfrac{\pi}{6}-\dfrac{1}{3}\doteq 0{,}19,\ f\left(\sqrt{3}\right)=\dfrac{\pi}{3}-3\doteq -1{,}95$.