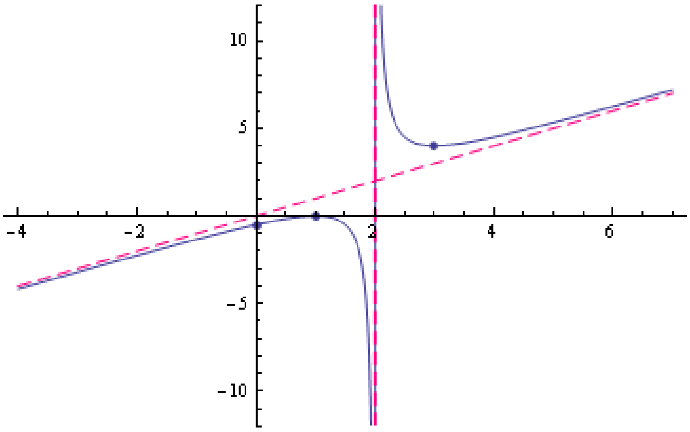
Behaviour of function

Investigate the behaviour of function *f*, if:

1. $f(x) = x + \dfrac{1}{x-2}$

Solution:

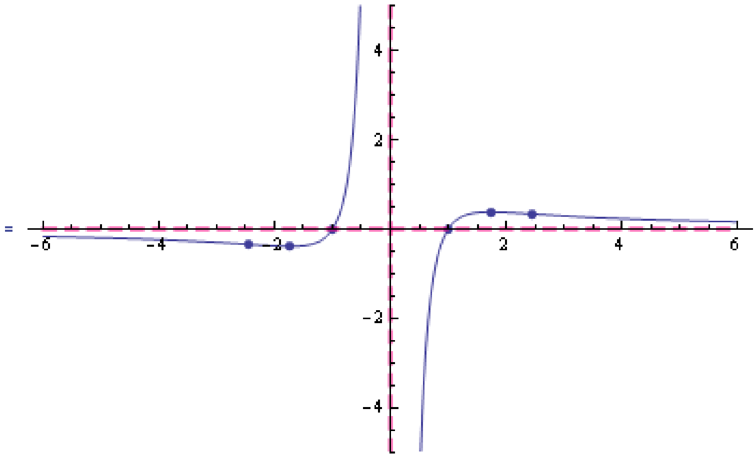
$D(f) = R - \{2\}$, zero point $x = 1$, point of discontinuity $x = 2$, function is not even, not odd, $x = 2$ is equation of asymptote without slope, $y = x$ is equation of asymptote with slope, function is increasing on $(-\infty,1\rangle$ *a* $\langle 3,\infty)$, decreasing on $\langle 1,2)$ *a* $(2,3\rangle$, local maximum is at the point $x = 1$ with value $f(1) = 0$, local minimum is at the point $x = 3$ with value $f(3) = 4$, function is convex on $(2,\infty)$, concave on $(-\infty,2)$, there are no inflex points, $H(f) = (-\infty,0\rangle \cup \langle 4,\infty)$.



2. $f(x) = \dfrac{x^2-1}{x^3}$

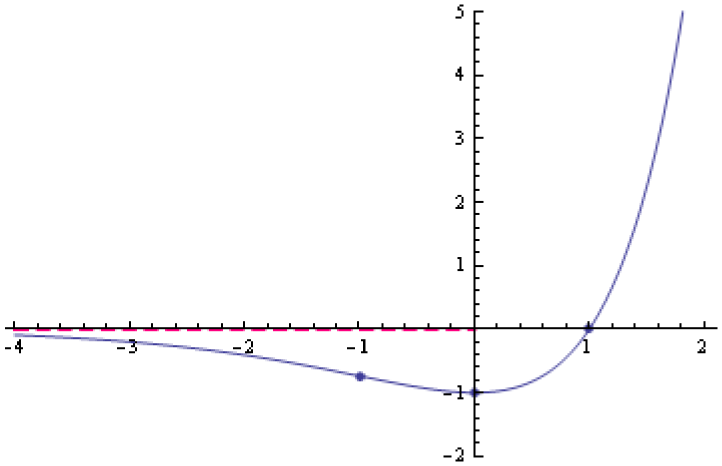
Solution:

$D(f) = R - \{0\}$,zero points $x = -1$, $x = 1$, points if discontinuity $x = 0$, function is odd, $x = 0$ is equation of asymptote without slope, $y = 0$ is equation of asymptote with slope, function is decreasing on $(-\infty,-\sqrt{3}\rangle$ *a* $\langle \sqrt{3},\infty)$, increasing on $\langle -\sqrt{3},0)$ *a* $(0,\sqrt{3}\rangle$, local maximum is at the point $x = \sqrt{3}$ with value $f(\sqrt{3}) = \dfrac{2}{3\sqrt{3}}$, locale minimum is at the point $x = -\sqrt{3}$ with value $f(-\sqrt{3}) = -\dfrac{2}{3\sqrt{3}}$, fnction is convex on $\langle -\sqrt{6},0)$ a $\langle \sqrt{6},\infty)$, concave on $(-\infty,-\sqrt{6}\rangle$ and $(0,\sqrt{6}\rangle$, $x = \pm\sqrt{6}$ are points of inflexion, $H(f) = R$.



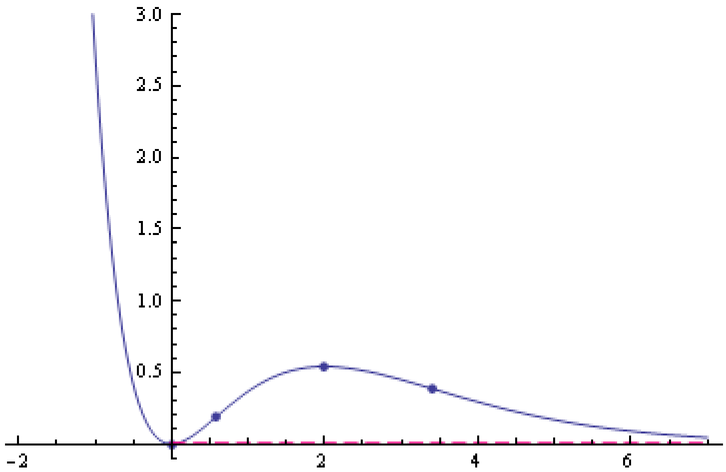
3. $f(x) = e^x(x-1)$

Solution:
$D(f) = R$, zero point is $x = 1$, function is not odd, nor even, it has no asymptote without slope, $y = 0$ is equation of asymptote with slope for $x \to -\infty$, function is increasing on $\langle 0,\infty)$, decreasing on $(-\infty,0\rangle$, local minimum is at the point $x = 0$ with value $f(0) = -1$, function is convex on $\langle -1,\infty)$, concave on $(-\infty,-1\rangle$, $x = -1$ is the point of inflexion, $H(f) = \langle -1,\infty)$.



4. $f(x) = \dfrac{x^2}{e^x}$

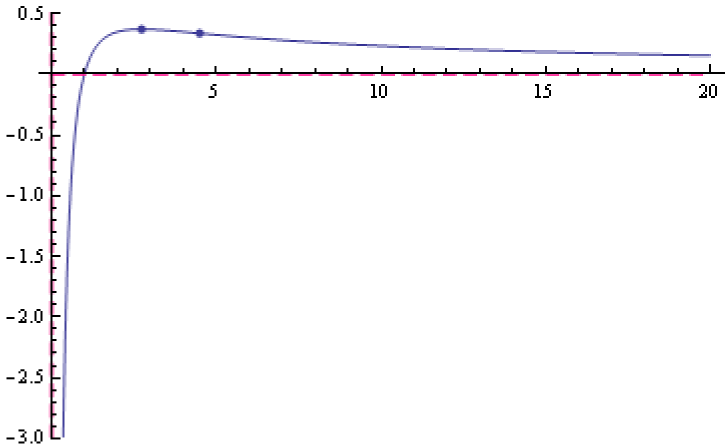
Solution:
$D(f) = R$, zero point is $x = 0$, function is not even, nor odd, it has no asymptotes without slope, $y = 0$ is equation of asymptote with slope for $x \to \infty$, function is increasing on $\langle 0,2\rangle$, i tis decreasing on $(-\infty,0\rangle$ a $\langle 2,\infty)$, it has local minimum at the point $x = 0$ with value $f(0) = 0$, local maximum at the point $x = 2$ with value $f(2) = \dfrac{4}{e^2}$, function is convex on $\left(-\infty,2-\sqrt{2}\right\rangle$ a $\left\langle 2+\sqrt{2},\infty\right)$, concave on $\left\langle 2-\sqrt{2},2+\sqrt{2}\right\rangle$, points $x = 2-\sqrt{2}$ a $x = 2+\sqrt{2}$ are inflexion points, $H(f) = \langle 0,\infty)$.



5. $f(x)=\dfrac{ln\,x}{x}$

Solution:

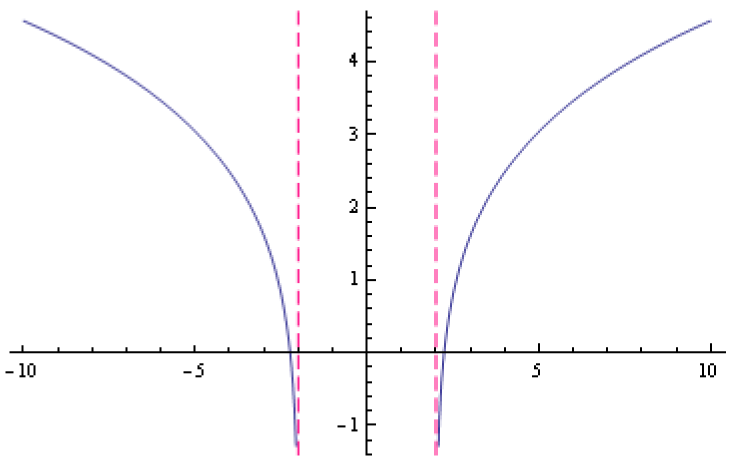
$D(f)=(0,\infty)$, zero point is $x=1$, function is not even, nor odd, $x=0$ is equation of asymptote without slope, $y=0$ is equation of asymptote with slope for $x\to\infty$, function is increasing on $\left(0,e\right\rangle$, decreasing on $\left\langle e,\infty\right)$, it has local maximum at the point $x=e$ with value $f(e)=\dfrac{1}{e}$, function is convex on $\left\langle\sqrt{e^3},\infty\right)$, concave on $\left(0,\sqrt{e^3}\right\rangle$, $x=\sqrt{e^3}$ is inflexion point, $H(f)=\left(-\infty,\dfrac{1}{e}\right\rangle$.



6. $f(x)=ln\left(x^2-4\right)$

Solution:

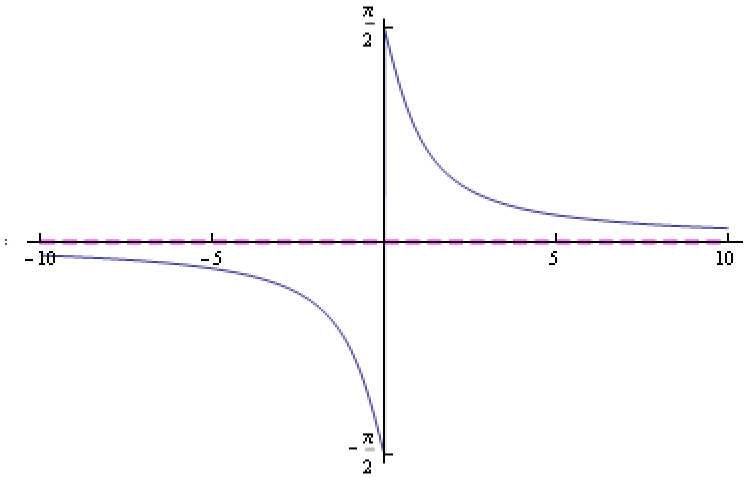
$D(f)=(-\infty,-2)\cup(2,\infty)$, zero points are $x=-\sqrt{5},\ x=\sqrt{5}$, function is even, $x=-2,\ x=2$ are equations of asymptotes without slope, function has no asymptotes with slope, i tis increasing on $(2,\infty)$, decreasing on $(-\infty,-2)$, there are no local extrema, function is concae on $(-\infty,-2)$ and $(2,\infty)$, there are no points of inflexion, $H(f)=R$.



7. $f(x) = arctg\,\frac{1}{x}$

Solution:

$D(f) = (-\infty,0) \cup (0,\infty)$, function has no zero points, one point of discontinuity $x = 0$, it is odd, it has no asymptotes without slope, $y = 0$ is equation of the asymptote with slope, function is decreasing on $(-\infty,0)$ and $(0,\infty)$, there are no local extrema, function is concave on $(-\infty,0)$, convex on $(0,\infty)$, there exist no points of inflexion, $H(f) = \left(-\frac{\pi}{2},0\right) \cup \left(0,\frac{\pi}{2}\right)$.



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