Systems of linear equations 2

System of *n* equations with *n* unknowns

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$$

is symbolically denoted in the matrix form

$$\mathbf{A.X} = \mathbf{B}$$

Suppose the inverse matrix $\mathbf{A}^{-1}$ to the matrix $\mathbf{A}$ exists. Multiple of the symbolic equation of the system from the left by inverse matrix $\mathbf{A}^{-1}$ gives

$$\mathbf{A}^{-1}.\mathbf{A}.\mathbf{X} = \mathbf{A}^{-1}.\mathbf{B}$$

$$\mathbf{E}.\mathbf{X} = \mathbf{A}^{-1}.\mathbf{B}$$

$$\mathbf{X} = \mathbf{A}^{-1}.\mathbf{B}$$

diagonal system of equations,

which has a unique solution, ordered *n*-tuple

$$(x_1, x_2, ..., x_n) = (\mathbf{A}^{-1}.\mathbf{B})^{T}$$

Original system of equations

and new diagonal system of equations

have the same solutions.

To solve given system of n linearly independent equations with n unknowns,

while rank of the matrix of the system is $h(\mathbf{A}) = n$,

the inverse matrix $\mathbf{A}^{-1}$ can be used,

which always exists to the regular matrix $\mathbf{A}$.

Calculation of inverse matrix to matrix A

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}\begin{pmatrix} D_{11} & D_{12} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2n} \\ \dots & \dots & \dots & \dots \\ D_{n1} & D_{n2} & \dots & D_{nn} \end{pmatrix}^T$$

where $D_{ij} = (-1)^{i+j} \, |\mathbf{A}_{ij}|$, $i, j = 1, 2, \dots, n$

is algebraic complement of element a_{ij} in matrix **A**,

determined by means of sub-determinant of matrix $\mathbf{A}_{ij}$,

which can be derived by eliminating i-th row and

j-th column of matrix **A.**