DERIVATIVE OF FUNCTION

Let function f be defined at the point x_0 and in some neighbourhood of this point. If there exists a limit

$$\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

then it is called the derivative of the function f at the point x_0 and denoted $f'(x_0)$.

For $\Delta x = x - x_0$, we receive

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

If the real number $f'(x_0)$ exists at the point x_0 , function f is said to be differentiable at the point x_0 .

If

$$\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \pm\infty$$

we speak about improper derivative of function f at the point x_0 , but f is not differentiable at x_0 .

One-sided derivatives of function at point

Real number

$$f'_{+}(x_0) = \lim_{\Delta x \to 0^+} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{x \to x_0^+} \frac{f(x) - f(x_0)}{x - x_0}$$

is derivative of function f on the right at the point x_0

$$f'_{-}(x_0) = \lim_{\Delta x \to 0^-} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{x \to x_0^-} \frac{f(x) - f(x_0)}{x - x_0}$$

is derivative of function f on the left at the point x_0 .

Function f has derivative at the point x_0 if and only if it has a derivative on the left and on the right at this point, and these are equal

$$f'(x_0) = f'_{+}(x_0) = f'_{-}(x_0)$$

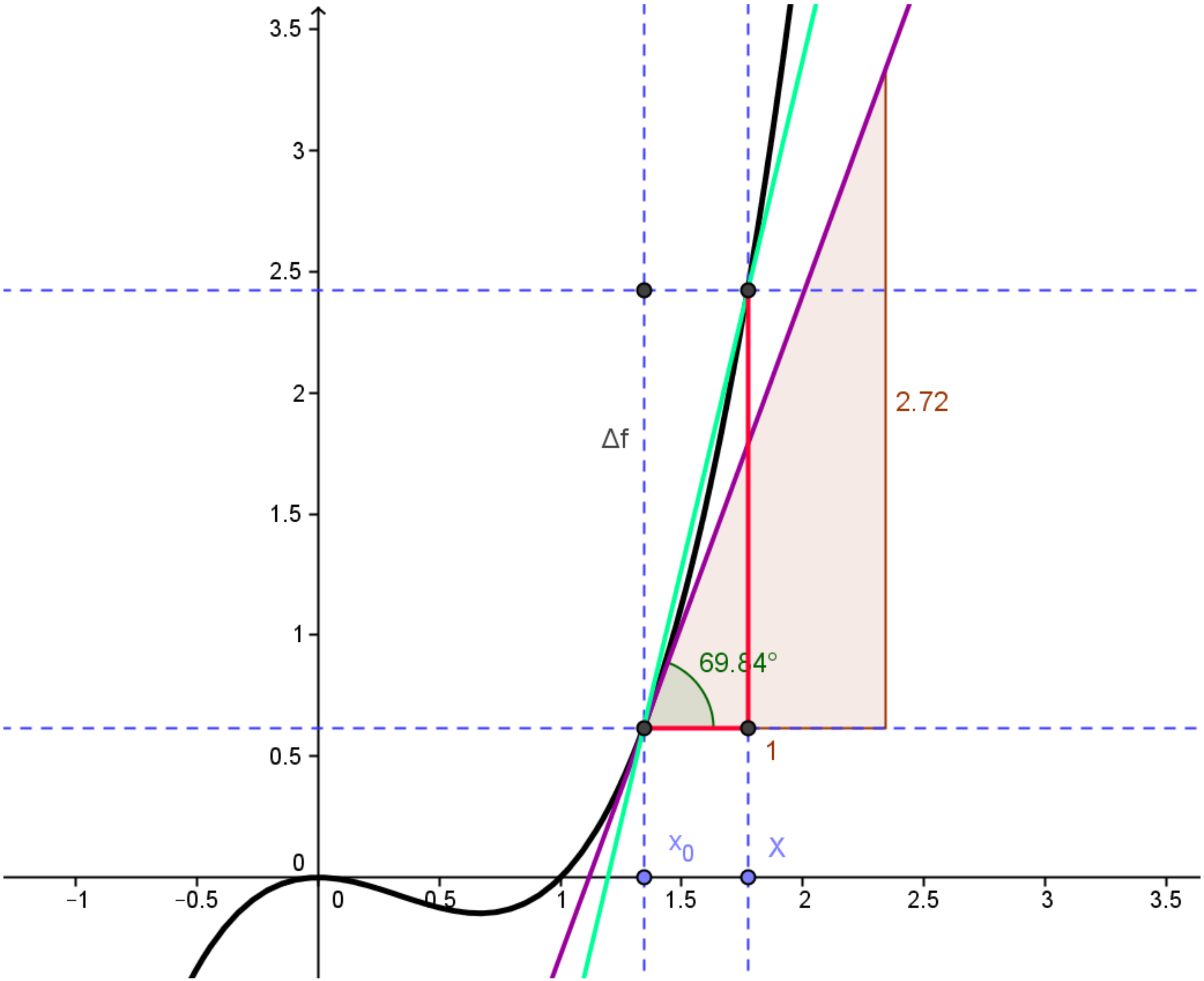
Geometric meaning of derivative

If $f'(x_0)$ is the derivative of function f at the point x_0, then tangent to the graph of function f at the point $T = [x_0, f(x_0)]$ has the slope equal to the number $f'(x_0)$ and equation of this tangent line is

$$y - f(x_0) = f'(x_0).(x - x_0)$$

If derivative $f'(x_0)$ of function f is improper and function f is continuous at the point x_0, then tangent line to the function graph at the point $T = [x_0, f(x_0)]$ is perpendicular to the coordinate axis x and its equation is $x = x_0$.

Geometric interpretation



Physical meaning of derivative

If a point moves along a straight line

and function $s = f(t)$ represents its law of motion where t = time, then

$s'(t_0)$ determines the value of this rectilinear motion velocity at time t_0

i.e. velocity of the point at the given moment.

If there exists derivative of function *f* at the point x_0, then function *f* is continuous at the point x_0. Function that is continuous at the point x_0 may have no derivative at the point x_0.

Derivative of function on a set

Let there exist derivatives of a function *f* at all points in the set *M*. Function *f'* , defined on *M,* in which any point $x_0 \in M$ is attached value $f'(x_0)$ is called derivative of function *f* on the set *M* and it is denoted

$$f'(x) = \frac{df(x)}{dx}$$

Basic rules for finding derivative

Let functions *f* and *g* have derivatives on the set *M*. Then also derivatives of the following functions exist on *M*

$$c.f,\ c\in R,\ f\pm g,\ f.g$$

and

$$1.\ [c.f(x)]' = c.f'(x)$$

$$2.\ [f(x)\pm g(x)]' = f'(x)\pm g'(x)$$

$$3.\ [f(x).g(x)]' = f'(x).g(x)+f(x).g'(x)$$

4.If $g(x) \neq 0$ for $\forall x \in M$,

then there exists also

derivative of function $\dfrac{f}{g}$ and

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x).g(x) - f(x).g'(x)}{g^2(x)}$$

Derivative of the composite function

Let there exist derivatives of functions $f(x)$ and $\varphi(x)$ at all points $x \in M$. Then there exists derivative of composite function $F(x) = f(\varphi(x))$ at all points $x \in M$, while

$$F'(x) = [f(\varphi(x))]' =$$

$$= [f'(u)]_{u=\varphi(x)} . \varphi'(x) = f'(\varphi(x)) . \varphi'(x)$$