FUNCTION CONTINUITY

Function *f* is continuous at the point *a*, if

$$\lim_{x \to a} f(x) = f(a)$$

therefore:

1. $a \in D(f)$
2. $\exists \lim_{x \to a} f(x)$
3. $\lim_{x \to a} f(x) = f(a)$

Function *f* is continuous on the left or on the right of the point *a* if

$$\lim_{x \to a^-} f(x) = f(a)$$

$$\lim_{x \to a^+} f(x) = f(a)$$

Function *f* is continuous on the set *M*, if it is continuous at all points in *M*.

Function f is continuous on a closed interval $\langle a, b \rangle$, if it is continuous at each point from the open interval (a, b) and it is continuous on the right at the point a and on the left at the point b.

If functions f and g are continuous at the point a, then also functions $f \pm g$ and $f \,.\, g$ are continuous at a. If $g(a) \neq 0$, then also function $f \,/\, g$ is continuous at a.

All elementary functions are continuous at each point of their domains of definition.

Let function *f* be not continuous at a point *a*, then the point is called a point of discontinuity of a function.

In such points function *f* :

1. is not defined, a $\notin$ *D*(*f*), but can have a limit

2. has no limit $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$

3. is defined and has a limit, but this is not equal to the function value

$$\lim_{x \to a} f(x) \neq f(a)$$

Properties of functions continuous on closed interval

1. Function *f* continuous on a closed interval is bounded on this interval.

2. A function *f* continuous on closed interval $\langle a, b\rangle$ assumes the greatest value (maximum) and the least value (minimum) in this interval

$$\exists c_1, c_2 \in \langle a, b \rangle, \textbf{\textit{such that}}$$

$$f(c_1) \le f(x) \le f(c_2) \forall x \in \langle a, b \rangle$$

3. If function f is continuous on closed interval $\langle a, b\rangle$ and $f(a)\,.\,f(b) < 0$, then there exists $c \in \langle a, b\rangle$, such that $f(c) = 0$.

4. If function f is continuous on closed interval $\langle a, b\rangle$, and it is not a constant function, then the image of interval $\langle a, b\rangle$ is again a closed interval $\langle c, d\rangle$.

Asymptotes of function graph

Line $y = k \, . \, x + q$ is called an asymptote to the graph of a function $f(x)$ with the slope,

if for $x \to \infty$, or for $x \to -\infty$, holds

$$k = \lim_{x \to \infty} \frac{f(x)}{x} \qquad q = \lim_{x \to \infty} (f(x) - k.x),$$

$$k = \lim_{x \to -\infty} \frac{f(x)}{x} \qquad q = \lim_{x \to -\infty} (f(x) - k.x)$$

Line $x = a$ is called asymptote to the graph of a function $f(x)$ without the slope, if at least one from the below one-sided limits at the point a is an improper limit, therefore at lest one from the following equalities is fulfilled:

$$\lim_{x \to a^+} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty$$

$$\lim_{x \to a^+} f(x) = -\infty \qquad \lim_{x \to a-} f(x) = -\infty$$

Line $x = a$ is an asymptote without the slope,

if function $f(x)$ is defined in the left or in the right neighbourhood of the point a

and it is not continuous at the point a.