SURVEY of METHODS
to SOLVE SELECTED TYPES of
DIFFERENTIAL EQUATIONS

1. LDE of order 1 with separated variables

$$p(x) + q(y)\,y' = 0$$

Solution – by integration of the equation

2. LDE of order 1 with separable variables

$$p_1(x)\,q_1(y) + p_2(x)\,q_2(y)\,y' = 0$$

Solution – by separation of variables

and integration of the equation

3. LDE of order 1 – homogeneous

$$y' + p(x)\, y = 0$$

Solution – by separation of variables and integration of the equation

$$y_h = c.e^{-\int p(x)dx}, c \in R$$

4. LDE of oder 1 – non-homogeneous

$$y' + p(x)\, y = q(x)$$

Solution – general solution of homogeneous LDE + particular solution of non-homogeneous LDE

$$y_v = y_h + \tilde{y} = c.e^{-\int p(x)dx} + c(x).e^{-\int p(x)dx}, c \in R, x \in R$$

$c(x)$ can be determined by method of variation of constant

5. LDE of order 2 – homogeneous

$$y'' + p_1 y' + p_2 y = 0$$

Solution – find fundamental system of solutions, i.e. 2 linearly independent particular solutions y_1, y_2 using roots of the characteristic equation

$$r^2 + p_1 r + p_2 = 0$$

a) $r_1 \neq r_2 \Rightarrow y_1 = e^{r_1 x}, y_2 = e^{r_2 x}$

b) $r = r_1 = r_2 \Rightarrow y_1 = e^{rx}, y_2 = xe^{rx}$

c) $r_1 = a + ib, r_2 = a - ib \Rightarrow y_1 = e^{ax} \cos bx, y_2 = e^{ax} \sin bx$

$$y_h = c_1 y_1 + c_2 y_2, \quad c_1, c_2 \in R$$

6. LDE of order 2 – non-homogeneous

$$y'' + p_1 y' + p_2 y = q(x)$$

Solution – find general solution of LDE homogeneous

1 particular solution of LDE non-homogeneous by method of variation of constants

$$y_v = y_h + \tilde{y} = c_1 y_1 + c_2 y_2 + \tilde{y}, \quad c_1, c_2 \in R$$

$$\tilde{y} = y_1 \int \frac{W_1}{W}\, dx + y_2 \int \frac{W_2}{W}\, dx$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \quad W_1 = \begin{vmatrix} 0 & y_2 \\ q(x) & y_2' \end{vmatrix} \quad W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & q(x) \end{vmatrix}$$

7. LDE of order 2 – non-homogeneous

$$y'' + p_1 y' + p_2 y = q(x)$$

Solution – find general solution of LDE homogeneous +

1 particular solution of non-homogeneous LDE by analysis of the special form the rigth-hand term $q(x)$

$$y_v = y_h + \tilde{y} = c_1 y_1 + c_2 y_2 + \tilde{y}, \quad c_1, c_2 \in R$$

a) $$q(x) = e^{\alpha x} P_n(x) \Rightarrow \tilde{y} = x^k e^{\alpha x} P_n^*(x), k \in \{0,1,2\}$$

b) $$q(x) = e^{\alpha x}(P_n(x)\cos\beta x + Q_m(x)\sin\beta x) \Rightarrow$$

$$\tilde{y} = x^k e^{\alpha x}(P_s^*(x)\cos\beta x + Q_s^*(x)\sin\beta x), k \in \{0,1\}$$