DIFFERENCIAL EQUATIONS of the FIRST ORDER

Linear differential equation of order one - LDE

is a differential equation in form

$$y' + p(x) \,.\, y = q(x)$$

where $p(x)$, $q(x)$ are continuous functions on interval (a, b).

If $q(x) = 0$ for $\forall x \in (a, b)$, then LDE is homogeneous, otherwise it is non-homogeneous.

General solution of LDE of order 1 in the form

$$y' + p(x) . y = 0$$

can be represented as

$$y_h = c.e^{-\int p(x)dx}, c \in R, x \in (a,b)$$

Steps to solve non-homogeneous LDE of order 1 in the form $y' + p(x) \cdot y = q(x)$

1. Find general solution of homogeneous LDE of order 1

$$y' + p(x) \cdot y = 0$$

$$y_h = c.e^{-\int p(x)dx}, c \in R, x \in (a,b)$$

2. Find one particular solution of given non-homogeneous LDE of order 1

$$y' + p(x) \,.\, y = q(x)$$

in the form

$$\tilde{y} = c(x).e^{-\int p(x)dx}, x \in (a,b)$$

using method of variation of constant,

where constant c from the general solution of homogeneous LDE 1 was exchanged by an unknown function $c(x)$.

Function $c(x)$ can be determnined by differentiating and inserting both $\tilde{y}, \tilde{y}'$

to the given non-homogeneous LDE of order 1.

3. General solution of non-homogeneous LDE of order 1 can be then represented in the form

$$y_G = y_h + \tilde{y}$$

$$y_h = c.e^{-\int p(x)dx}, c \in R, x \in (a,b)$$

$$\tilde{y} = c(x).e^{-\int p(x)dx}, x \in (a,b)$$