DIFERENTIAL EQUATIONS
of the FIRST ORDER
Ordinary differential equation with
SEPARATED VARIABLES
is differential equation in the form

\[ p(x) + q(y)y' = 0 \]

where \( p(x) \) is continuous function on interval \((a, b)\)
and \( q(y) \) is continuous function on interval \((c, d)\).

Any solution of ODE

\[ p(x) + q(y)y' = 0 \]

is in the form

\[ \int p(x)\,dx + \int q(y)\,dy = c, c \in \mathbb{R} \]

**Note:** If \( q(y) \neq 0 \) on interval \((c, d)\), then exactly one integral
curve is passing through any point of set \( D = (a, b) \times (c, d) \).
Ordinary differential equation with SEPARABLE VARIABLES

is differential equation in the form

\[ p_1(x) \cdot q_1(y) + p_2(x) \cdot q_2(y) \ y' = 0 \]

where \( p_1(x) \), \( p_2(x) \) are continuous functions on \((a, b)\)
and \( q_1(y) \), \( q_2(y) \) are continuous functions on \((c, d)\).

If \( p_2(x) \cdot q_1(y) \neq 0 \), then ODE is equivalent with ODE

\[
\frac{p_1(x)}{p_2(x)} + \frac{q_2(y)}{q_1(y)} \ y' = 0
\]
Solution of ODE

\[ p_1(x) \cdot q_1(y) + p_2(x) \cdot q_2(y) \cdot y' = 0 \]

are all functions defined by equation

\[
\int \frac{p_1(x)}{p_2(x)} \, dx + \int \frac{q_2(y)}{q_1(y)} \, dy = c, \quad c \in R
\]

and constant functions (singular solutions)

\[ y_i = b_i, \quad \text{pre} \ q_1(y_i) = 0 \]