

DIFFERENTIAL EQUATIONS

Basic concepts

Ordinary differential equation = ODE

is an equation representing relation between variable x from some set $M \subset \mathbb{R}$, an unknown function $f(x)$ and at least one from derivatives $f'(x), f''(x), \dots, f^{(n)}(x)$, $n \in \mathbb{N}$ of this function.

Denoting $f(x) = y$, $f'(x) = y'$, $f''(x) = y''$, $\dots$, $f^{(n)}(x) = y^{(n)}$, symbolic representation of differential equation can be written

$$V(x, y, y', y'', \dots, y^{(n)}) = 0$$

If at most n -th derivative of unknown function, i.e. $y^{(n)}$ appears in ODE and there appears no derivative of a higher order, equation is called ordinary differential equation of order n .

All derivatives of function $y = f(x)$ of lesser orders than n need not explicitly appear in an ODE of n -th order, nor function f itself and even not the variable x .

Solution of ODE of order n on a non-empty set $M \subset \mathbb{R}$ is any function $y = \varphi(x)$ defined on M and possessing there all derivatives up to order n , while for all $x \in M$ it satisfies the equation.

To solve a differential equation means to find all functions satisfying given equation and to define set $M = (a, b) \subset \mathbb{R}$, on which all solutions are defined, or alternatively, to show that given equation has no solution.

While solving differential equations we distinguish the following types of solutions:

1. General solution

in which the number of present coefficients equals to the order of the equation

2. Particular solution

which can be obtained from the general solution by a suitable choice of constants, or these coefficients can be derived from the initial conditions

3. Singular solution

which contains no constant and cannot be obtained from general solution

Method for solving solution is called also **integration of differential equation**, function that is the solution is called **integral of differential equation** and graph of this solution is called **integral curve of differential equation**.

Two differential equations are **equivalent**, if any solution of one of them is also solution of the other one on the same set.

While solving differential equations we can use the following operations:

1. equivalent manipulations

Original and resulting equations are equivalent, they have the same solutions.

2. non-equivalent manipulations

Resulting equation can have more or less solutions than the original equation, or the set, on which solutions are defined can be different. All solutions therefore must be separately analyzed.

Problem to find particular solution of ODE of the first order satisfying initial condition - **Cauchy initial condition**

$y(x_0) = y_0$, where x_0, y_0 are given numbers,
is called **Cauchy initial problem**.

Geometric interpretation

Integral curve is passing through the point $Q = [x_0, y_0]$.

Problem to find particular solution of ODE of order 2 satisfying initial conditions - **Cauchy initial conditions**

$y(x_0) = y_0, y'(x_0) = y_1$, where x_0, y_0, y_1 are given numbers,
is called **Cauchy initial problem**.

Geometric interpretation

Integral curve is passing through the point $Q = [x_0, y_0]$
and it has a tangent line at this point with the slope y_1 .