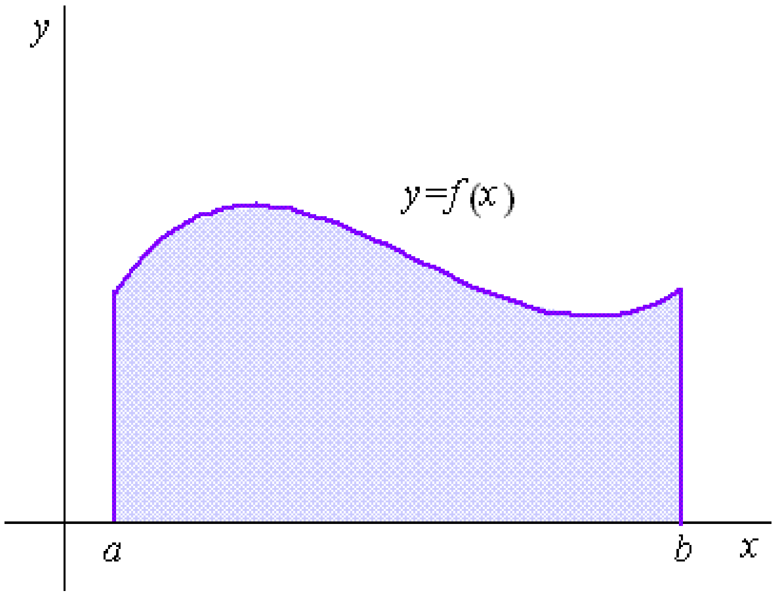
GEOMETRIC APPLICATIONS OF DEFINITE INTEGRALS

Area of curvilinear trapezoid

Let function $f(x)$ be continuous and non-negative on $J = \langle a, b \rangle$.



$$KL = \{[x, y] \in R^2; a \le x \le b, 0 \le y \le f(x)\}$$

$$P(KL) = \int_a^b f(x)dx$$

Area of curvilinear trapezoid

Let function $f(x) \leq 0$ for all $x \in [a, b]$, then

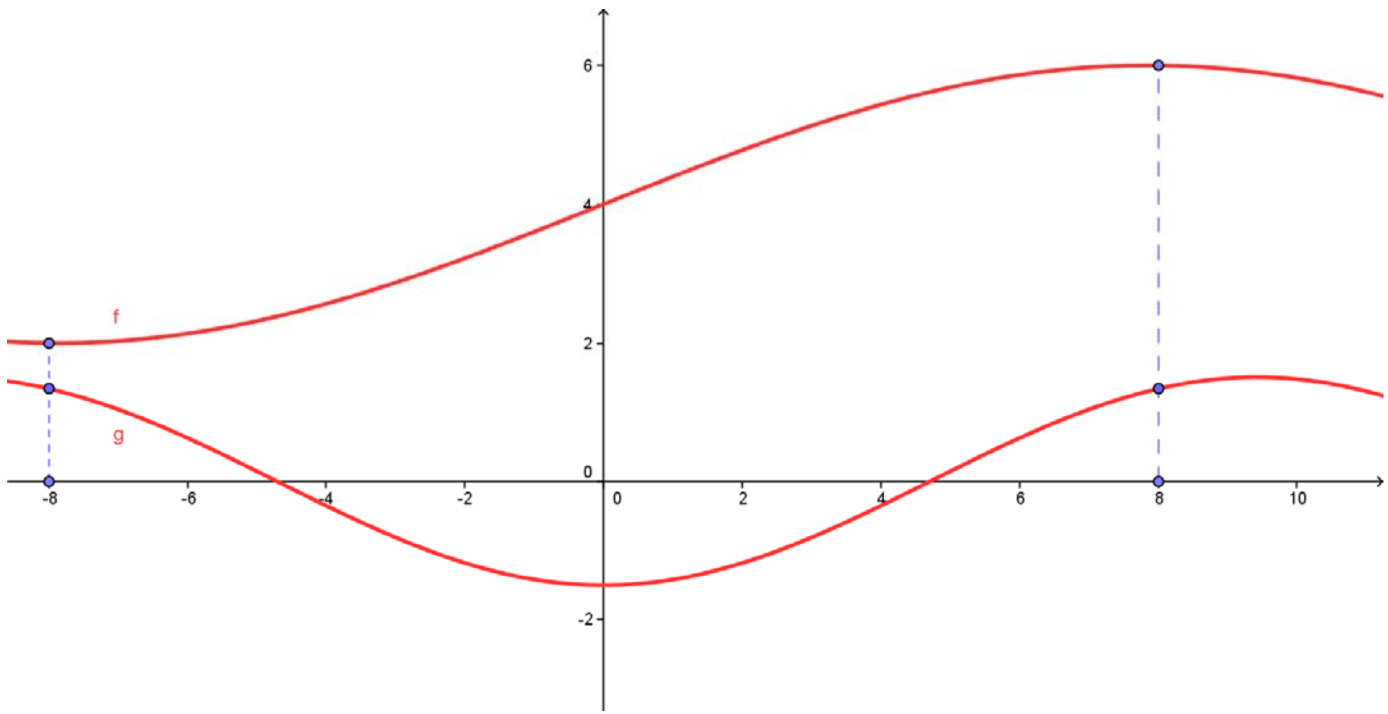
$$\int_a^b f(x)dx \leq 0$$

and area of respective curvilinear trapezoid can be calculated by formula

$$P(KL) = \left| \int_a^b f(x)dx \right|$$

Area of elementary region

Let functions $f(x)$ and $g(x)$ be continuous on $J = \langle a, b \rangle$.

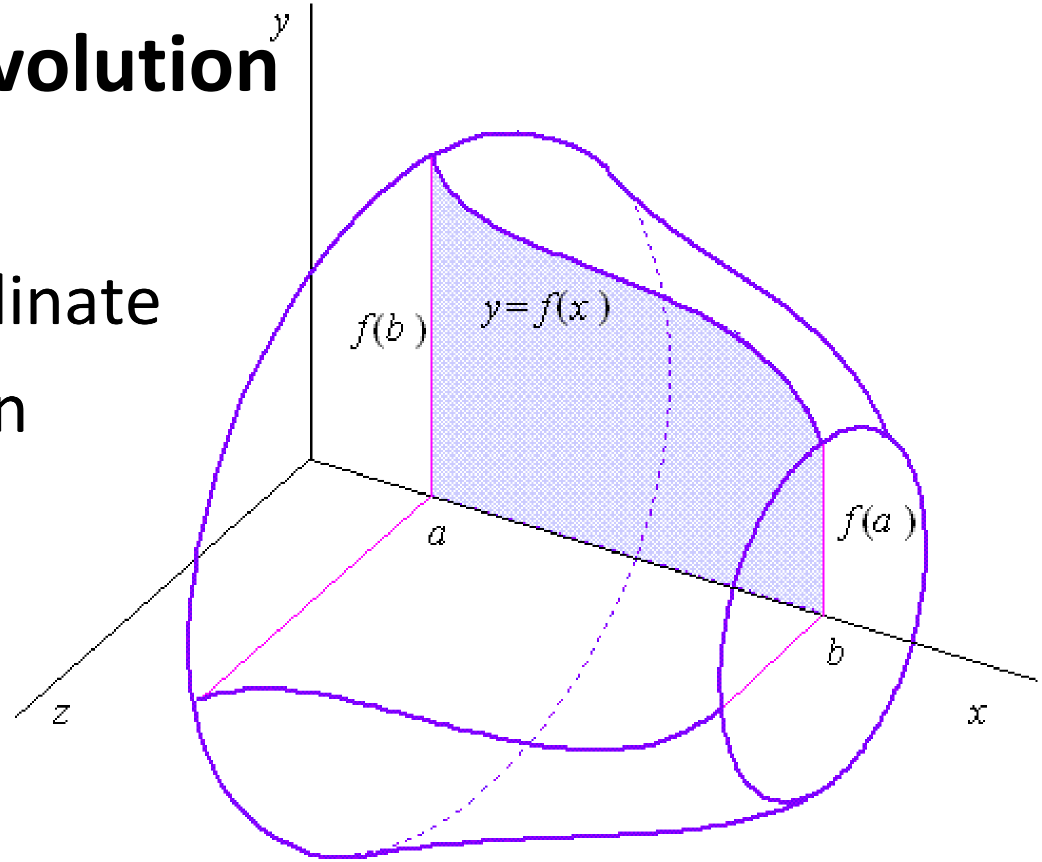


$$EO = \{[x, y] \in R^2; a \le x \le b, g(x) \le y \le f(x)\}$$

$$P(EO) = \int_a^b (f(x) - g(x))dx$$

Volume of solid of revolution

Revolving curvilinear trapezoid *KL* about coordinate axis *x* a solid of revolution Is generated.



$$KL = \{[x, y] \in R^2; a \le x \le b, 0 \le y \le f(x)\}$$

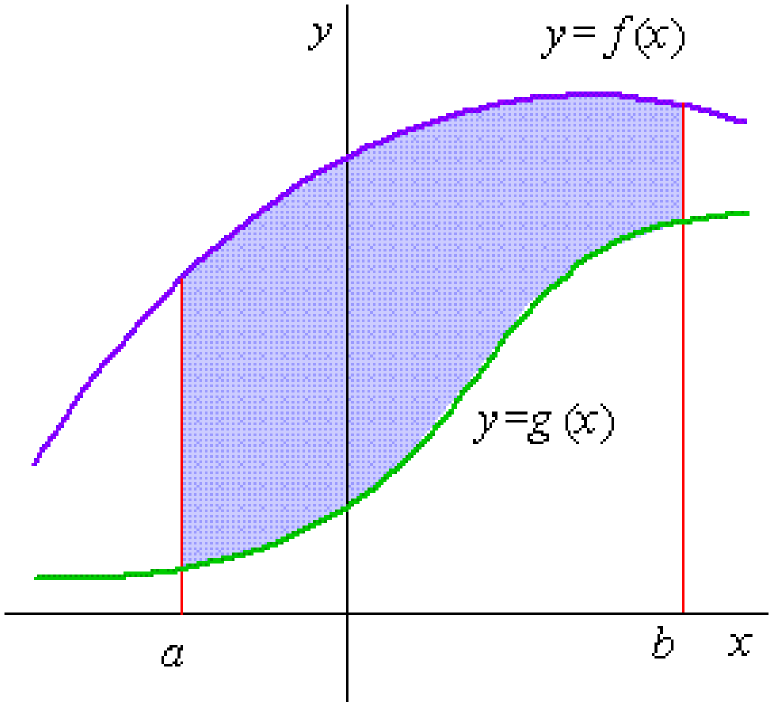
$$V = \pi \int_a^b f^2(x) dx$$

Volume of solid of revolution

Revolving elementary region *EO*

about coordinate axis *x*

a solid of revolution is generated

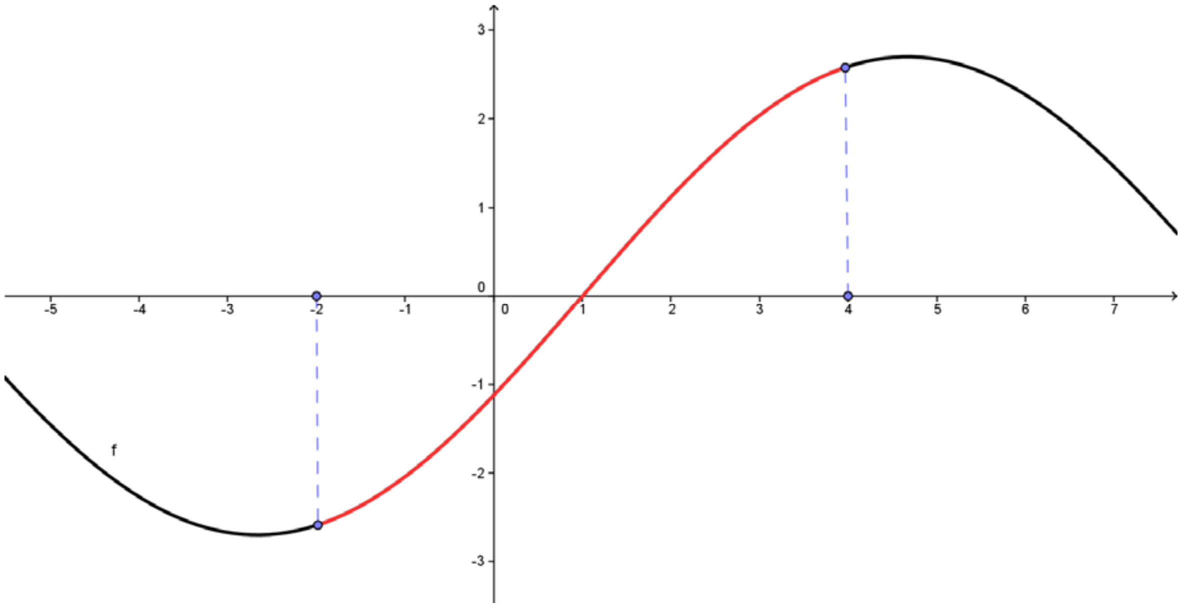


$$EO = \{[x, y] \in R^2; a \le x \le b, g(x) \le y \le f(x)\}$$

$$V = \pi \int_a^b (f^2(x) - g^2(x))dx$$

Length of planar curve segment

Let a planar curve segment be graph of function *f*(*x*) continuous on $J = \langle a, b \rangle$ and let continuous derivative $f'(x)$ of function exists on *J*

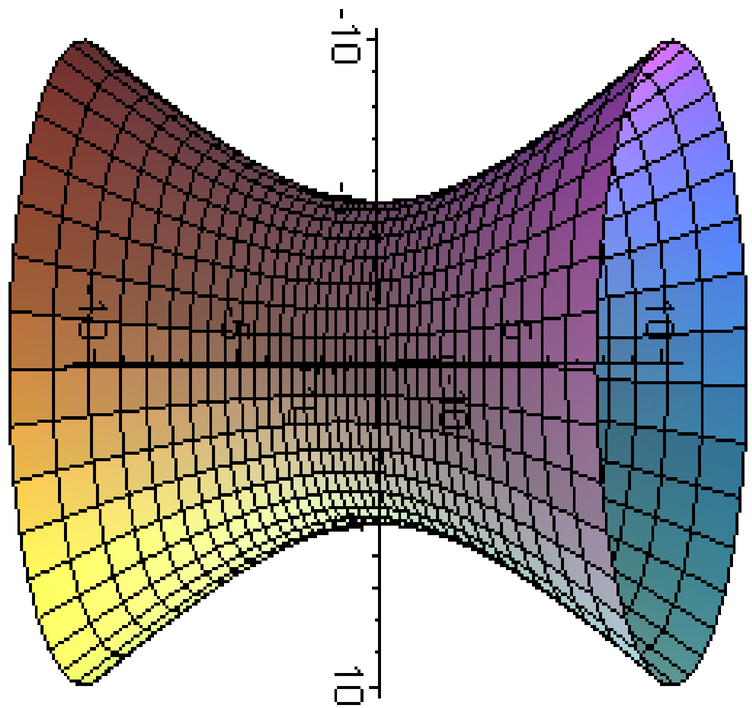


$$K = \{[x, y] \in R^2; a \le x \le b, y = f(x)\}$$

$$D(K) = l = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Area of surface of revolution

Revolving planar curve segment *K* about coordinate axis *x* a surface of revolution is generated



$$K = \{[x, y] \in R^2; a \le x \le b, y = f(x)\}$$

$$S = 2\pi \int_a^b |f(x)| \sqrt{1 + (f'(x))^2}\, dx$$

PHYSICAL APPLICATIONS OF DEFINITE INTEGRALS

Mass of a solid of revolution

Let solid of revolution be generated by revolution about axis x of a curvilinear trapezoid determined by function $y = f(x)$ on interval $\langle a, b\rangle$. Then, if ρ is the specific density of the solid material, solid mass of the can be calculated as

$$m = \rho.V = \rho\pi\int_a^b f^2(x)dx$$

Static moment of a solid of revolution

Static moment with respect to axis *x* can be calculated as

$$S_x = \rho\pi\int_a^b x.f^2(x)dx$$

Centre of gravity of a solid of revolution

Center of gravity is the point

$T = [x_T, 0, 0]$, while

$$x_T = \frac{S_x}{m} = \frac{\int_a^b x.f^2(x)dx}{\int_a^b f^2(x)dx}$$

Moment of innertia of a solid of revolution

Moment of innertia can be calculated as

$$J = \frac{\rho\pi}{2}\int_a^b f^4(x)dx$$

Kinetic energy of a solid of revolution

Kinetic energy can be calculated with respect to the revolutionary movement with the angular velocity ω as

$$E = \frac{\rho \pi \omega^2}{2} \int_a^b f^4(x) dx$$