# DEFINITE INTEGRAL

Let $J = \langle a, b \rangle$ be closed interval on set **R**.
Ordered set of points

$$D = \{x_0 , x_1, \ldots, x_n\}, \text{ kde } \; a = x_0 < x_1 < \ldots < x_n = b$$

is called division of interval *J.*

Interval $\langle x_{i\text{-}1} , x_i \rangle$ is called *i*-th partial interval,
$\Delta x_i = x_i - x_{i\text{-}1}$ is the length of this partial interval

$$\xi_i \in \langle x_{i\text{-}1} , x_i \rangle \Rightarrow f(\xi_i) \Rightarrow f(\xi_i).\Delta x_i \Rightarrow \sum_{i=1}^{n} f(\xi_i).\Delta x_i$$

**Definite integral**

Let function $f(x)$ be bounded on interval $J = \langle a, b \rangle$.

If for any selection of points $\xi_i$ there exists the same proper limit

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i$$

then the limit is called definite integral of function $f(x)$ on interval $J = \langle a, b \rangle$ and it is denoted

$$\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i$$

Function $f(x)$ is called integrable on interval $J$.

**The following rules hold:**

1. $$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

2. $$\int_a^a f(x)dx = 0$$

## Basic properties of definite integral

1. Function $f(x)$ that is continuous on interval $J = \langle a, b \rangle$, is integrable on this interval.

2. Function $f(x)$ that is bounded on interval $J = \langle a, b \rangle$, while it has just a finite number of points of discontinuity on this interval, is integrable on this interval.

3. Let functions $f(x)$ and $g(x)$ be integrable on interval $J = \langle a, b\rangle$ and let $c \in \langle a, b\rangle$ , $k \in R$.

Then the following hold:

a) $$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

b) $$\int_a^b k.f(x)dx = k.\int_a^b f(x)dx$$

c) $$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

4. If for all $x \in \langle a, b\rangle$ holds $f(x) \geq 0$, then

$$\int_a^b f(x)dx \geq 0$$

5. If for all $x \in \langle a, b\rangle$ holds $f(x) \leq g(x)$, then

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx$$

## Evaluation of definite integral

**Newton-Leibniz formula**

Let functions $f(x)$ and $F(x)$ be continuous on interval $J = \langle a, b\rangle$ and let function $F(x)$ be antiderivative of function $f(x)$ on interval $(a, b)$. Then the following holds:

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

## Basic integration methods for definite intregrals

### 1. Integration by Parts (Per Partes Methods)

Let $u(x)$ and $v(x)$ be functions with continuous derivatives $u'(x)$ and $v'(x)$ defined on intervale $J$.

Then on interval $J$ holds

$$\int_a^b u(x)v'(x)dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)dx$$

# Basic integration methods for definite intregrals

## 2. Substitution method

If the function integrated on interval $\langle a, b\rangle$ can be represented in the form $f(\varphi(x)).\varphi'(x)$, where $\varphi(x)$ and $\varphi'(x)$ are continuous functions on interval $\langle a, b\rangle$ and function $f(x)$ is continuous at all points $t = \varphi(x)$, $x \in \langle a, b\rangle$, then holds

$$\int_a^b f(\varphi(x)).\varphi'(x)dx = \int_{\varphi(a)}^{\varphi(b)} f(t)dt$$

## Some basic properties

1. Let $f(x)$ be an even function, then

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

2. Let $f(x)$ be an odd function, then

$$\int_{-a}^{a} f(x)dx = 0$$

## Mean value of function on interval

Let function $f(x)$ be integrable on interval $\langle a, b \rangle$, then number

$$\mu = \frac{1}{b-a} \int_a^b f(x)dx$$

is the mean value of function $f(x)$ on interval $\langle a, b \rangle$.

Let $f(x)$ be a continuous function on interval $\langle a, b \rangle$, then there exists at least one number $c \in \langle a, b \rangle$ such, that

$$\mu = f(c) = \frac{1}{b-a} \int_a^b f(x)dx$$