INDEFINITE INTEGRAL

Let $J = (a, b)$ be open interval in real numbers **R**.

Function $F(x)$ is called

antiderivative (primitive function)

of function $f(x)$ on interval J

if and only if for all x from interval J holds

$$F'(x) = f(x).$$

The following theorems hold:

1. Let function $F(x)$ be antiderivative of function $f(x)$ on interval J, then also function $F(x) + c$ is antiderivative of function $f(x)$ on interval J for any real number c.

2. Let functions $F(x)$ and $G(x)$ be antiderivatives of function $f(x)$ on interval J. Then there exists such real number c, that for all x from interval J hold

$$G(x) = F(x) + c$$

3. Let function $f(x)$ be continuous on interval J, then there exists an antiderivative of $f(x)$ on interval J.

Indefinite integral

Set of all antiderivatives of function $f(x)$ on interval J is called indefinite integral of function $f(x)$, which is denoted

$$\int f(x)dx$$

To integrate a function means to find all its antiderivatives .

If $\forall x \in J : F'(x) = f(x) \Rightarrow$

$$\int f(x)dx = F(x) + c, \forall x \in J, c \in R$$

Real number c is called integration constant, integrated function is called integrand, expression dx determines integration variable.

Properties of indefinite integrals:

1. $$\int f'(x)dx = f(x) + c, \forall x \in J, c \in R$$

2. $$[\int f(x)dx]' = f(x), \forall x \in J$$

Basic integration formulas

1. $\int x^{\alpha} dx = \dfrac{x^{\alpha+1}}{\alpha+1} + c, \alpha \neq -1, \alpha \in R$

2. $\int \dfrac{1}{x} dx = \ln|x| + c$

3. $\int e^{x} dx = e^{x} + c$

4. $\int a^{x} dx = \dfrac{a^{x}}{\ln a} + c, a > 0, a \neq 1$

5. $\int \sin x dx = -\cos x + c$

6. $\int \cos x dx = \sin x + c$

7. $\int \frac{1}{\cos^2 x} dx = tgx + c$

8. $\int \frac{1}{\sin^2 x} dx = -\cot gx + c$

9. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + c, a > 0$$

10. $\int \frac{1}{1+x^2} dx = arctgx + c$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} arctg \frac{x}{a} + c, a > 0$$

$$11.\ \int \frac{1}{\sqrt{x^2 \pm a^2}}\, dx = \ln\left| x + \sqrt{x^2 \pm a^2} \right| + c$$

$$12.\ \int \frac{f'(x)}{f(x)}\, dx = \ln\left| f(x) \right| + c$$

All formulas hold on any open interval $J = (a, b)$, which is the subset of domain of definition of the integrated function.

Basic properties of indefinite integrals

If there exist antiderivatives of functions $f(x)$ and $g(x)$ on interval J, then also functions

$f(x) \pm g(x)$ and $k. f(x)$, $k \in \mathbf{R}$

have antiderivatives on interval J, while:

1. $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

2. $\int k.f(x)dx = k.\int f(x)dx$

Basic integration methods

1. Per Partes Method – Integration by Parts

Let $u(x)$ and $v(x)$ be functions, that are differentiable on interval J, while their derivatives $u'(x)$ and $v'(x)$ are continuous.

Then on interval J holds

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

Basic integration methods

2. Substitution method

Let $F(t)$ be antiderivative of function $f(t)$ on interval (a, b). Let function $\varphi(x)$ be differentiable on interval (α, β) while $\varphi'(x)$ be its continuous derivative and let for all $x \in (\alpha, \beta)$ be $\varphi(x) \in (a, b)$. Then holds

$$\int f(\varphi(x))\varphi'(x)dx = F(\varphi(x)) + c,\ x \in (\alpha, \beta)$$

Symbolic notation

1.

$$\int f(\varphi(x))\varphi'(x)dx = \left| \begin{array}{c} t=\varphi(x) \\ dt=\varphi'(x)dx \end{array} \right| =$$

$$= \int f(t)dt = F(t)+c = F(\varphi(x))+c,\ x \in (\alpha, \beta)$$

2.

$$\int f(x)dx = \left| \begin{array}{c} x=\varphi(t) \Rightarrow t=\varphi^{-1}(x) \\ dx=\varphi'(t)dt \end{array} \right| =$$

$$= \int f(\varphi(t))\varphi'(t)dt = F(t)+c = F(\varphi^{-1}(x))+c$$

if there exists $\varphi^{-1}(x)$.