CONVEXITY, CONCAVITY
POINTS OF INFLECTION
FUNCTION BEHAVIOUR

Convexity, concavity of function

Let $f(x)$ be continuous on interval J and let there exists second derivative at each interior point of this interval.

If for all points x from interior of interval J holds

$$f''(x) > 0$$

we say that function f is convex on interval J,

if for all x from interior of interval J holds

$$f''(x) < 0$$

we say that function f is concave on interval J.

Consequence

For coordinates [x, y] of arbitrary point on tangent line to function graph at point [x_0 , y_0] holds

$$f(x) > y = y_0 + f'(x_0)(x - x_0) \text{ for interval of convexity}$$

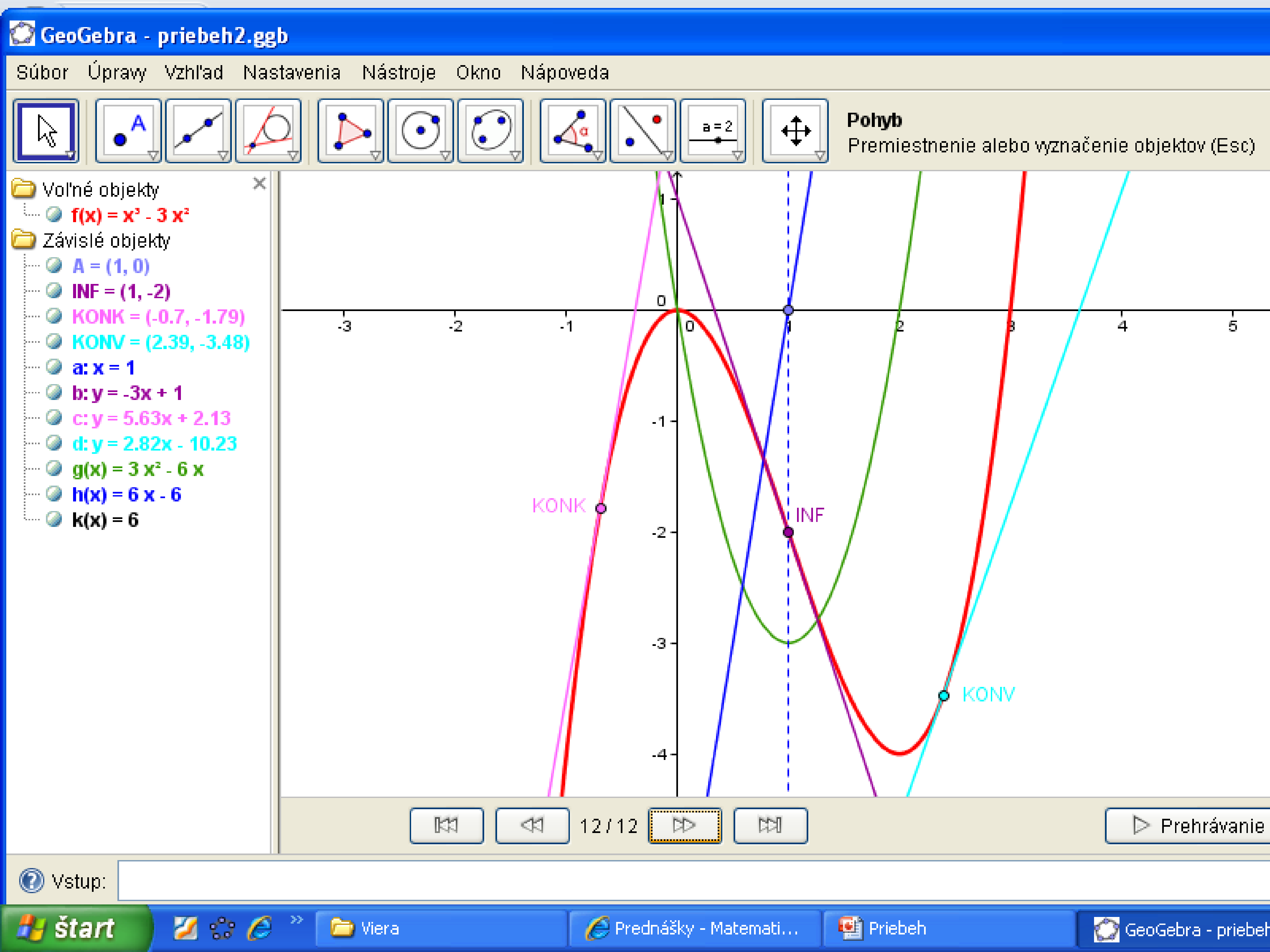
$$f(x) < y = y_0 + f'(x_0)(x - x_0) \text{ for interval of concavity}$$

Geometric interpretation

Function f continuous on interval $J \subset R$, which has the first derivative f' at all interior points of interval J,

is convex (concave) on interval J,

if for any tangent line to the function graph holds that all points of the graph but the tangent point are located over (below) this tangent line.


GeoGebra - priebeh2.ggb
Súbor Úpravy Vzhľad Nastavenia Nástroje Okno Nápoveda
Pohyb
Premiestnenie alebo vyznačenie objektov (Esc)
Voľné objekty
f(x) = x³ - 3 x²
Závislé objekty
A = (1, 0)
INF = (1, -2)
KONK = (-0.7, -1.79)
KONV = (2.39, -3.48)
a: x = 1
b: y = -3x + 1
c: y = 5.63x + 2.13
d: y = 2.82x - 10.23
g(x) = 3 x² - 6 x
h(x) = 6 x - 6
k(x) = 6
KONK
INF
KONV
12 / 12
Prehrávanie
Vstup:
Štart
Viera
Prednášky - Matemati...
Priebeh
GeoGebra - priebeh

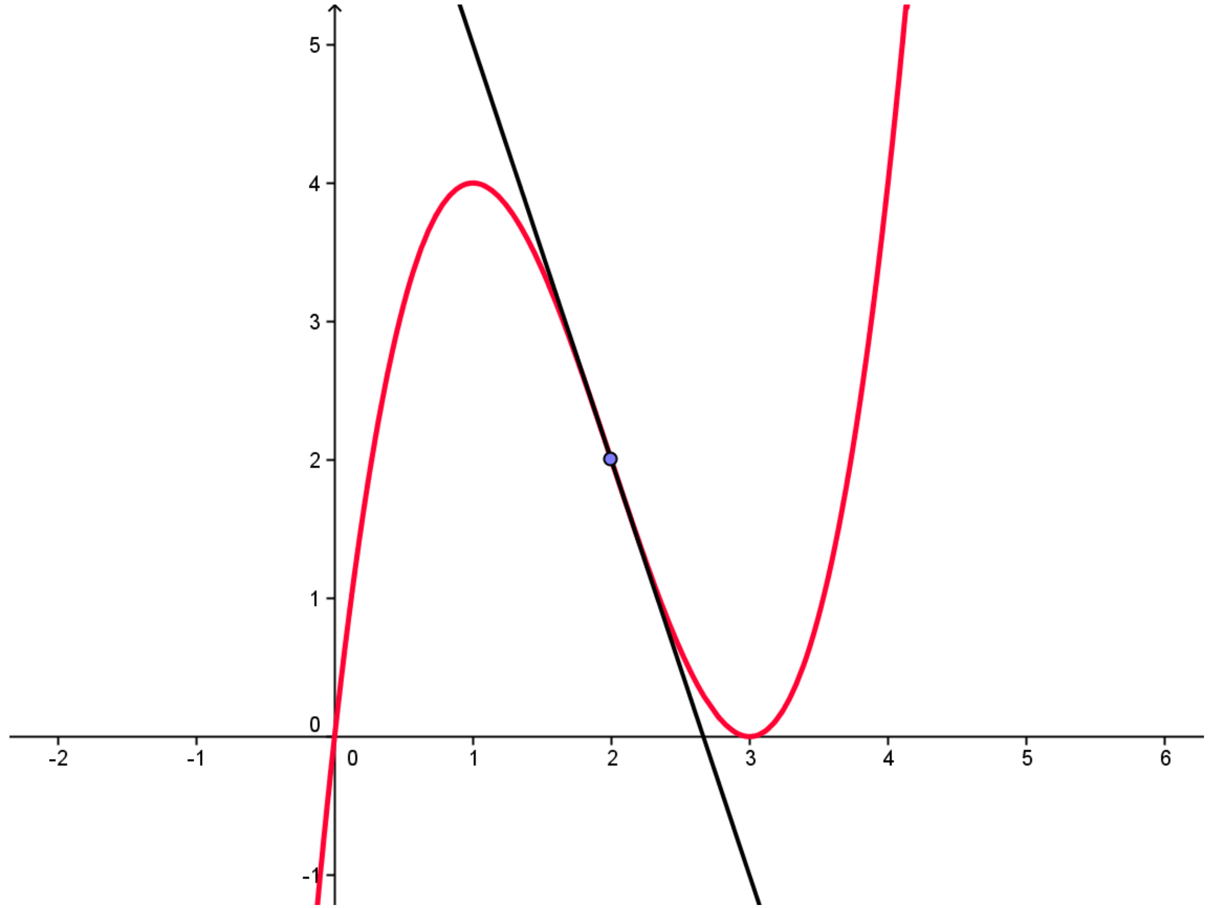
Point of inflection

Point x_0 , at which there exists derivative of function *f*, is called point of inflection, if function *f* is convex (concave) on some left-hand neighbourhood of point x_0 and it is concave (convex) on some right-hand neighbourhood of point x_0 .

If point x_0 is point of inflection of function *f*, then point $P_0 = [x_0, f(x_0)]$ on the function graph is the point of inflection of the function graph.

Geometric interpretation

Tangent line intersects function graph at the point of inflection, the graph is crossing from one half-plane determined by tangent line to the opposite half-plane.



Sufficient condition

for the existence of point of inflection

Let there exists the third non-zero derivative, $f^{(3)}(x_0) \neq 0$ of function f at point x_0 and let $f''(x_0) = 0$.

Then point x_0 is the point of inflection of function f.

Let $f'(x_0) = f''(x_0) = \ldots = f^{(n-1)}(x_0) = 0$, but $f^{(n)}(x_0) \neq 0$.

If n is even number, then point x_0 is the point of strict local extremum

maximum, if $f^{(n)}(x_0) < 0$

minimum, if $f^{(n)}(x_0) > 0$.

If n is an odd number, then point x_0 is point of inflection of function f.

Guides for investigation of function behaviour

1. Determine domain of definition, find points of discontinuity and zero points.
2. Inspect parity/non-parity of function, and its periodicity.
3. Determine asymptotes to function graph.
4. Determine intervals, on which function is monotone, and find points that are points of function local (global) extrema.
5. Find points if inflection and determine intervals, on which function is convex and concave.
6. Calculate coordinates of some points on the function graph – compose table of function values.
7. Sketch function graph.