DERIVATIVES OF HIGHER ORDERS

Let f be a differentiable function on set M

and let its derivative f' has a derivative at all points in M.

This derivative is called **second derivative** of function f or derivative of the second order and it is denoted f''

$$f''(x) = [f'(x)]' \quad \text{for all } x \in M.$$

Derivative of order *n*

Let function f has on set M derivative of order (n-1), for $n \in N$. If there exists derivative of the derivative of order (n-1) of function f for all $x \in M$, then this is called **derivative of order *n***, or the ***n*-th derivative** of function f, and we denote it $f^{(n)}$

$$f^{(n)}(x) = [f^{(n-1)}(x)]' \quad \text{for all } x \in M.$$

Differential of a function

Let f be a function defined and differentiable on some neighbourhood of point x_0 .

Number

$$\Delta f = f(x) - f(x_0)$$

is called **difference** of function f for increment

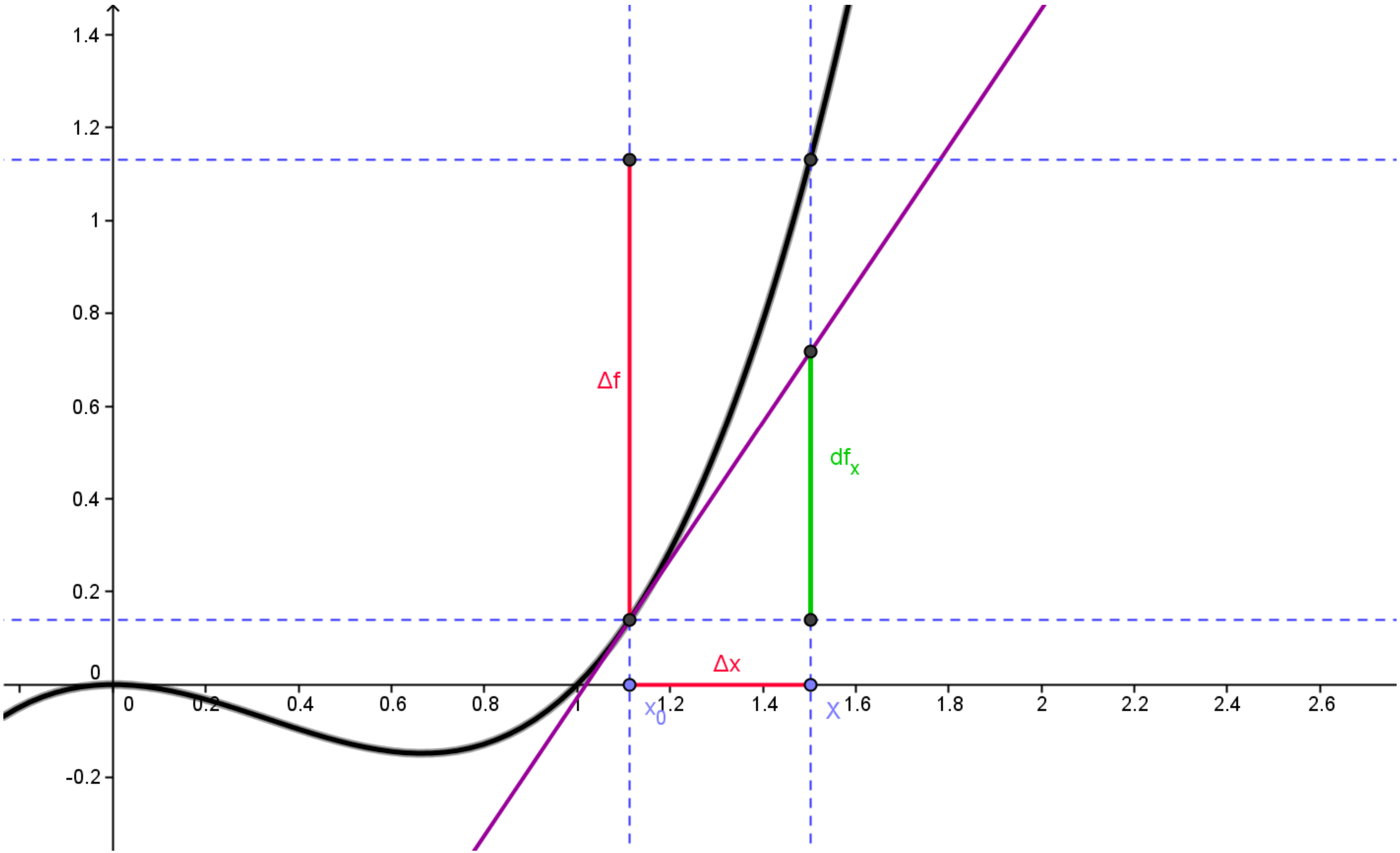
$$\Delta x = x - x_0$$

Formula $f'(x_0)(x-x_0)$

is called **differential** of function at point x_0 and it is denoted

$$df_{x_0}(x) = f'(x_0)(x - x_0)$$

Geometric interpretation of function differential



Taylor polynomial

Let there exist derivatives of function *f* at the point x_0 up to the order *n*, for $n \in N$.

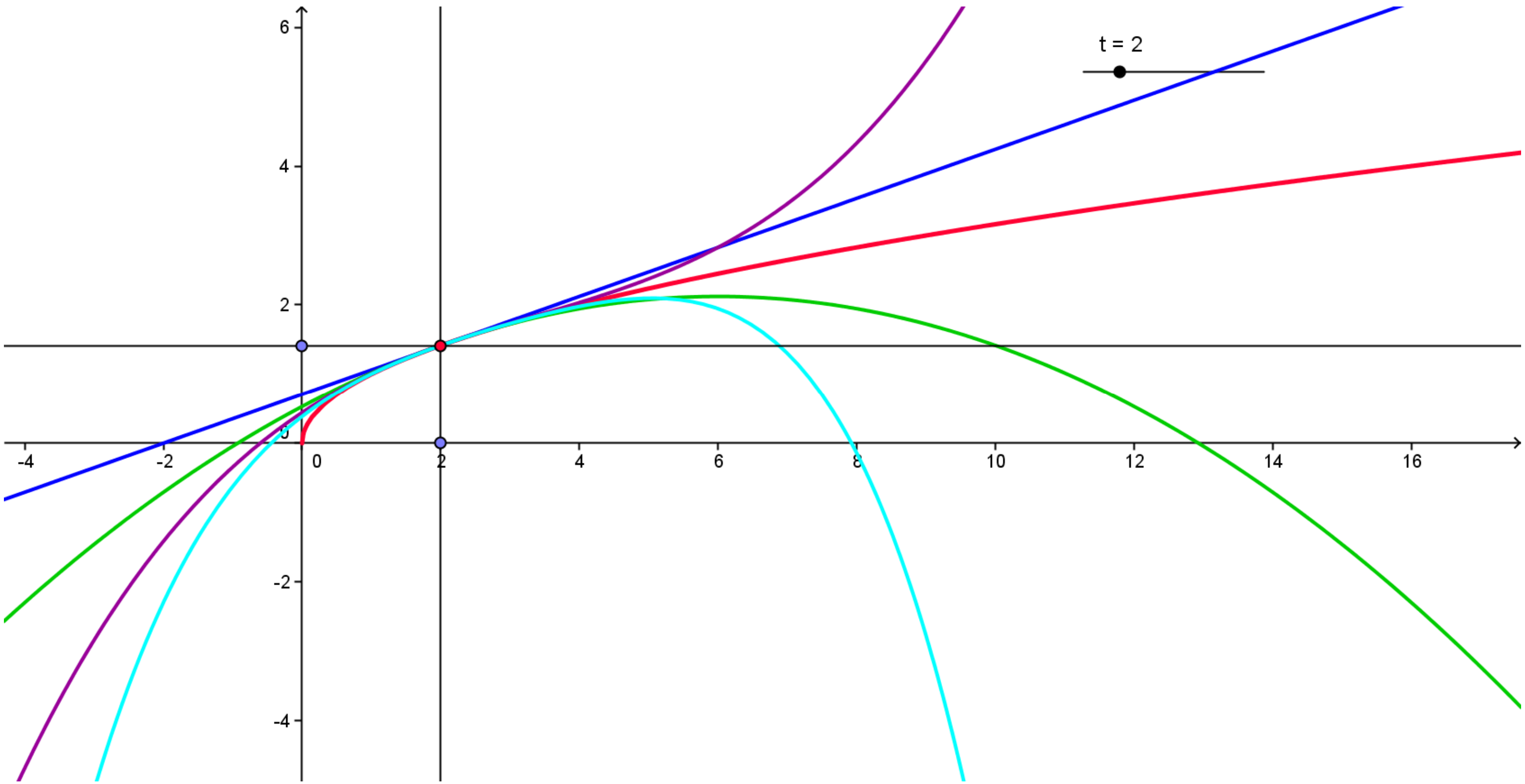
Polynomial

$$T_n(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots\ldots\ldots\ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n = \sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!}(x - x_0)^i$$

is called *n*-th Taylor polynomial of function *f* at the point x_0, while

$$f^{(0)}(x_0) = f(x_0), \;\; 0! = 1.$$

Geometric interpretation



Function value at point

$$f(x) = \sqrt{x} \qquad x_0 = 4$$

$$f(4{,}1) = 2{,}024845673131658$$

$$T_1(x) = 2 + \frac{x-4}{4}$$

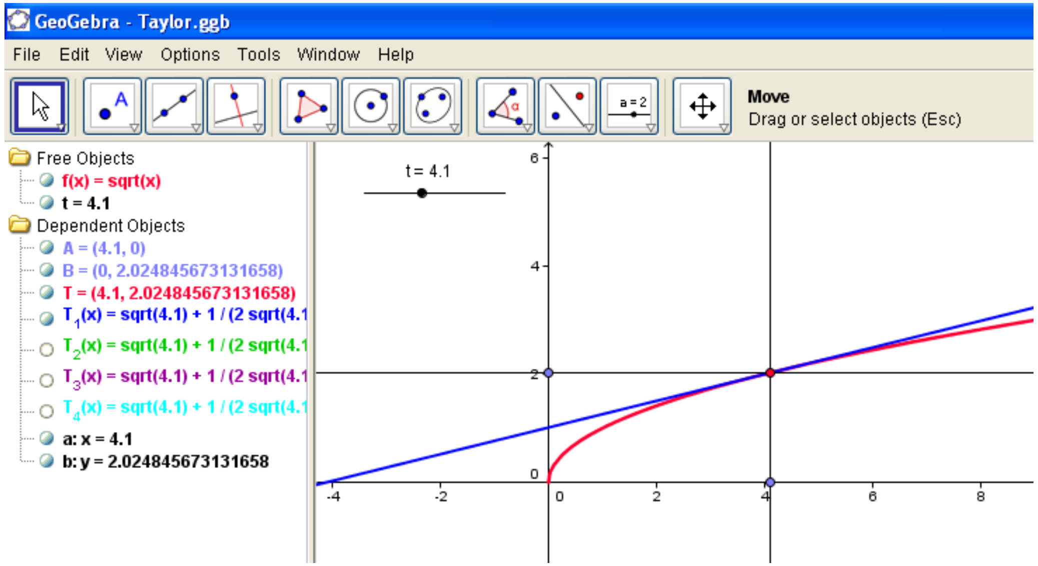
$$T_1(4{,}1) = 2{,}025$$

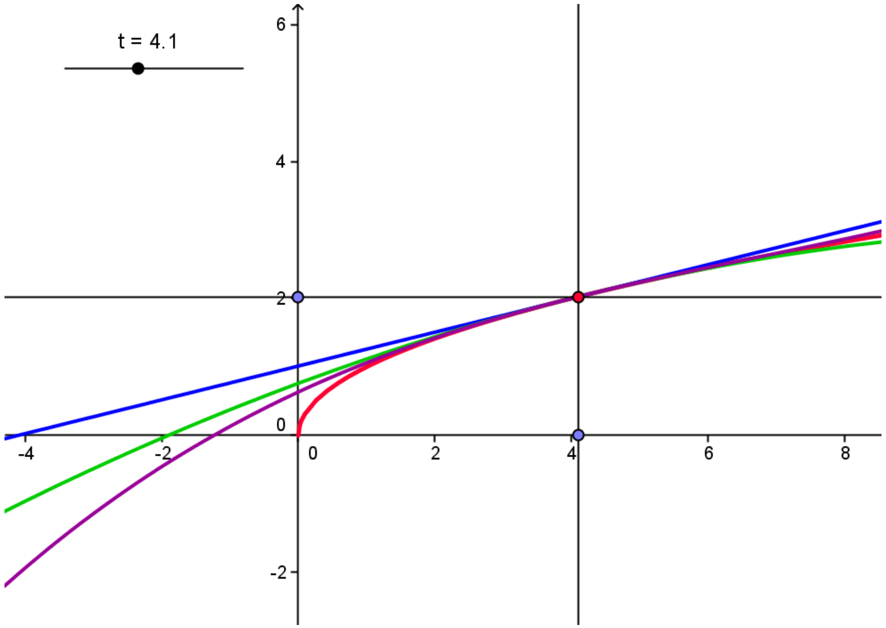
$$T_2(x) = 2 + \frac{x-4}{4} - \frac{(x-4)^2}{64}$$

$$T_2(4{,}1) = 2{,}02484375$$

$$T_3(x) = 2 + \frac{x-4}{4} - \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512}$$

$$T_3(4{,}1) = 2{,}024845703125$$





L´Hospital rule

can be used for evaluation of limits leading to one from the undetermined expressions of type

$$\frac{0}{0} \quad \frac{\infty}{\infty} \quad 0.\infty \quad \infty - \infty \quad 0^0 \quad 1^\infty \quad \infty^0$$

L´Hospital rule

Let $$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$$

or $$\lim_{x\to a} |f(x)| = \lim_{x\to a} |g(x)| = \infty$$

and let there exists proper or improper limit

$$\lim_{x\to a} \frac{f'(x)}{g'(x)}$$. Then there exists also $$\lim_{x\to a} \frac{f(x)}{g(x)}$$

while $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Remarks

1. Rule is valid also for one-sided limits and limits at improper points.
2. Rule can be reused multiple-times, provided all assumptions are fulfilled.