Applications of multiple integrals

1. Evaluate area of a planar region *M*:

a) $M:(x-2)^2+y^2\le 4, y\ge x$ b) $M:x^2+y^2\le 1, y\ge x$

c) $M:1\le x^2+y^2\le 9, y\ge 0$ d) $M:1\le x^2+y^2\le 4, x\ge 0, y\ge 0$

e) $M:y=5-x^2, \frac{x^2}{4}+\frac{(y-3)^2}{9}=1$ f) $M:y=(x-3)^2, y=\sqrt{x}$

Solution: a) (2 + π)/2 b) π/2 c) 4π d) 3π/4 e) 12, 3851 f) 3,05778

2. Calculate mass of the plate *M* from the Problem 1 for the density function:

a) $\mu(x,y)=xy$ b) $\mu(x,y)=\sqrt{x^2+y^2}$ c) $\mu(x,y)=\sqrt{1+x^2+y^2}$

d) $\mu(x,y)=\sqrt{x^2+y^2-1}$ e) $\mu(x,y)=x+y+1$ f) $\mu(x,y)=y\ln x$

Solution: a) 7/12 b) π/6 c) $\frac{4}{3}\sqrt{63-5\sqrt{5}}\pi$ d) √3π/2 e) 40,9167 f) 3,75389

3. Find coordinates of the centre of gravity of the plate *M* from the Problem 1 for the respective density function from the Problem 2.

Solution: a) $T=\left[\frac{9}{7},\frac{12}{7}\left(\frac{2}{15}+\frac{\pi}{16}\right)\right]$ b) $T=\left[0,\frac{3}{2\pi}\right]$ c)*T* = [0, 1.4515]

d) *T* = [1.05358, 1.05358] e) *T* = [0.205244, 2.75374]...f) T = [3.34574, 1.3149]

4. Calculate surface area of a part of surface that is graph of function *f*(*x*, *y*) on region *M* :

a) *f*(*x*, *y*) = 4 - x^2 - y^2, *M*: $0\le z\le f(x,y)$ b) $f(x,y)=\sqrt{x^2+y^2}, 1\le z\le 2$

c) *f*(*x*, *y*) = 8 - x^2 - y^2, *M*: $1\le x^2+y^2\le 4, \ |x|\le y$ ..

d) $f(x,y)=\sin\sqrt{x^2+y^2}, M:4\le x^2+y^2\le 9$

e) $f(x,y)=xy, M:0\le y\le 5-x^2$

f) $f(x,y)=x^3+y^3, M:\frac{x^2}{4}+\frac{y^2}{9}\le 1$

Solution: a) π(17√17 – 1)/6 b) √2 π c) π(17√17 – 5√5)/24 d) 20,1073 e) 69,337 f) 163,459

5. Evaluate volume of a solid *T*:

a) $T:0\le z\le 3, x^2+y^2\le 4$ b) $T:x^2+y^2=1, z=0, z=4-x-y$

c) $T:x^2+y^2=4, z=4, z=1-x^2-y^2$ d) $T:x^2+y^2=9, z=0, z=\sqrt{x^2+y^2}$

e) $T:0\le x^2+y^2+z^2\le 4$ f) $T:x^2+y^2+z^2\le 1, x\ge 0, y\ge 0, z\ge 0$

g) $T:1\le x^2+y^2+z^2\le 4$ h) $T:x^2+y^2+z^2\le 1, z\le x^2+y^2$

ch) $0\le z\le 1-(x^2+y^2), x^2+y^2\le 1$ i) $x^2+y^2+z^2\le 4, x^2+y^2-z^2\ge 0$

Solution: a) 12π b) 4π c) 20π d) 18π e) 32π/3 f) π/6 g) 28π/3 h) 7π/6 ch) π/2 i) 16π/3

6. Calculate mass of a solid T from the Problem 5 for the density function

a) $\mu(x,y,z)=x^2+y^2$ b) $\mu(x,y,z)=\sqrt{x^2+y^2}$ c) $\mu(x,y,z)=x+1$

d) $\mu(x,y,z)=y+1$ e) $\mu(x,y,z)=x^2+y^2+z^2$ f) $\mu(x,y,z)=xyz$

g) $\mu(x,y,z)=x+y+z$ h) $\mu(x,y,z)=-z$ ch) $\mu(x,y,z)=z^2$ i) $\mu(x,y,z)=z$

Solution: a) 24π b) 8π/3 c) 20π d) 18π e) 128π/5

f) 1/48 g) 45π/2 h) π/12 ch) π/12 i) 8(4-2√2)

7. Find coordinates of the centre of gravity of the solid T from the Problem 5 for the respective density function from the Problem 6.

Solution: a) $T=\left[0,0,\frac{3}{2}\right]$... b) $T=\left[-\frac{3}{40},-\frac{3}{40},\frac{83}{40},\right]$ c) $T=\left[\frac{17}{15},0,\frac{41}{30},\right]$

d) $T=\left[0,\frac{27}{10},\frac{9}{8}\right]$ e) T = [0, 0, 0] f) $T=\left[\frac{16}{35},\frac{16}{35},\frac{16}{35}\right]$

g) $T=\left[\frac{248(4+\pi)}{675\pi},\frac{248(4+\pi)}{675\pi},\frac{248(4+\pi)}{675\pi}\right]$ h) $T=\left[0,0,\frac{13}{5}\right]$

ch) $T=\left[0,0,\frac{3}{5}\right]$ i) T = [0, 0, 0]