Multiple integrals

1. Evaluate double integrals:

a) $\iint_M x^2 y\,dxdy, M: x=2, x=5, y=1, y=3$ b) $\iint_M xy\,dxdy, M: x=0, y=0, x+y=1$

c) $\iint_M \cos x\,dxdy, M: y=0, y=\sin x, x\in\left\langle 0, \frac{\pi}{2}\right\rangle$ d) $\iint_M 2y\,dxdy, M: x=0, y=2-x, y=\sqrt{x}$

e) $\iint_M 2x\,dxdy, M: y=\sin x, y=-x, x=\pi$ f) $\iint_M (2x+y)\,dxdy, M: y=0, y=1-x, y=1+x$

g) $\iint_M \frac{x}{y^2}\,dxdy, M: y=x^2, y=4x^2, x\ge 0, y=4$

Solution: a) 156 b) 1/24 c) ½ d) 11/6 e) 2π +2π/3 f) 1/3 g) 3ln 2/4

2. By means of polar transformation evaluate double integrals:

a) $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dxdy$ b) $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} e^{-(x^2+y^2)}dxdy$ c) $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} \operatorname{arctan}\frac{y}{x}\,dxdy$ d) $\int_{0}^{\pi}\int_{0}^{\sqrt{\pi^2-x^2}} \sin\sqrt{x^2+y^2}\,dxdy$

e) $\iint_M (1-2x-3y)dxdy, M$ is disc with radius 3 and centre in the origin of coordinates

f) $\iint_M \sin\sqrt{x^2+y^2}\,dxdy, M: \pi^2 \le x^2+y^2 \le 4\pi^2$

g) $\iint_M \sqrt{4-(x^2+y^2)}\,dxdy, M: x^2+y^2 \le 2x$

h) $\iint_M \frac{\ln(x^2+y^2)}{(x^2+y^2)}dxdy, M: 1\le x^2+y^2 \le e, x\ge 0, y\ge 0$

ch) $\iint_M 2-\sqrt{x^2+y^2}\,dxdy, M: x^2+y^2 \le 4, y\ge |x|$

i)) $\iint_M (x^2+y^2)dxdy, M: (x-1)^2+y^2 \le 1, y\ge 0$

j) $\iint_M \sqrt{x^2+y^2}\,dxdy, M: x^2+(y-1)^2 \le 1$

Solution: a) π/2 b) π(1-e^{-1})/2 c) π^2/16 d) 2π(π - 1) e) 9π
f) -6π^2 g) 8(3π - 4) /9 h) π/4 ch)) 2π/3 i) 3π/4 j) 32/9

3. Evaluate triple integrals:

a) $\iiint_T x^2y^2z\,dxdydz, T: x=1, x=3, y=0, y=2, z=2, z=5$

b) $\iiint_T (xy^2+z-2)\,dxdydz, T: x=0, x=3, y=0, y=1, z=0, z=4$

c) $\iiint_T (2y+1)\,dxdydz, T: x=0, x=2, y=0, y=2, z=0, x+y-z=0$

d) $\iiint_T xyz\,dxdydz, T: y=x, y=2x, y=2, z=0, z=4$

e) $\iiint\limits_T (x-y+2)dxdydz, T: 0 \le x \le 1, o \le y \le 1-x, 4x^2 \le z \le 5-x^2$

f) $\iiint\limits_T 3dxdydz, T: -\sqrt{4-y^2} \le x \le \sqrt{4-y^2}, 0 \le y \le 2, 0 \le z \le 2x+y$

g) $\iiint\limits_T (2x+3y-z)dxdydz, T: x=0, y=0, x+y=1, z=0, z=2$

h) $\iiint\limits_T (x-y+2z)dxdydz, T: x=1, y=0, x+y=3, z=0, 3y+z=6$

ch) $\iiint\limits_T \frac{dxdydz}{(x+y+z+1)^3}, T: x=0, y=0, z=0, x+y+z=1$

i) $\iiint\limits_T xyz\,dxdydz, T: x \ge 0, y \ge 0, z \ge 0, x^2+y^2+z^2 \le 1$

j) $\iiint\limits_T y\cos(x+z)dxdydz, T: 0y \le \sqrt{x}, z=0, x+z=\frac{\pi}{2}$

Solution: a) 728/3 b) 6 c) 69 d) 12 e) 4 f) 16
g) 2/3 h) 46 ch) (ln 2-5/8)/2 i) 1/48 j) π^2/16-1/2

4. By means of cylindrical and spherical coordinates evaluate triple integrals:

a) $\iiint\limits_T (x^2+y^2)z\,dxdydz, T: 0 \le z \le 3, x^2+y^2 \le 4$

b) $\iiint\limits_T \sqrt{x^2+y^2}\,dxdydz, T: x^2+y^2=1, z=0, z=4-x-y$

c) $\iiint\limits_T x\,dxdydz, T: x^2+y^2=4, z=0, z=1+x^2+y^2$

d) $\iiint\limits_T (y+1)\,dxdydz, T: x^2+y^2=9, z=0, z=\sqrt{x^2+y^2}$

e) $\iiint\limits_T (x^2+y^2+z^2)\,dxdydz, T: x^2+y^2+z^2 \le 4$

f) $\iiint\limits_T (x+2z)\,dxdydz, T: 1 \le x^2+y^2+z^2 \le 9, z \ge 0$

g) $\iiint\limits_T xyz\,dxdydz, T: x^2+y^2+z^2 \le 1, x \ge 0, y \ge 0, z \ge 0$

h) $\iiint\limits_T \arctan\frac{y}{x}\,dxdydz, T: x^2+y^2 \le 1, x \ge 0, y \ge 0, 0 \le z \le 2$

ch) $\iiint\limits_T \sin\sqrt{x^2+y^2}\,dxdydz, T: x^2+y^2 \le \pi^2, 0 \le z \le x^2+y^2$

i) $\iiint\limits_T dxdydz, T: |x+y| \le 1,\ 0 \le z \le 1-(x^2+y^2), x^2+y^2 \le 1$

j) $\iiint_T (y+1)\,dxdydz, T: x^2+y^2=9, 0\le z\le \sqrt{x^2+y^2}$

k) $\iiint_T x\,dxdydz, T: x^2+y^2=4, z=0, z=1+x^2+y^2$

l) $\iiint_T (x+y+z)\,dxdydz, T: 1\le x^2+y^2+z^2\le 4$

m) $\iiint_T z\,dxdydz, T: x^2+y^2+z^2\le 1, z\le x^2+y^2$

Solution: a) 36π b) $8\pi/3$ c) 0 d) 18π e) $128\pi/5$ f) 40π g) 1/48 h) $\pi^2/8$
ch) $2\pi^2(\pi^2 - 6)$ i) $\pi/4 - 2/3$ j) 18π k) 12π l) $4\pi - \pi/4$ m) $\pi(\pi/16+1/8)$