

Functions of two variables

1. Determine and sketch domain of definition of function $f(x, y) = \ln(xy)$. Describe properties of $D(f)$. Does point $A = [-1, 0]$ belong to $D(f)$? Is A a boundary point of $D(f)$?

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : (x < 0 \wedge y < 0) \vee (x > 0 \wedge y > 0)\}$, $A \notin D(f)$, A is boundary point

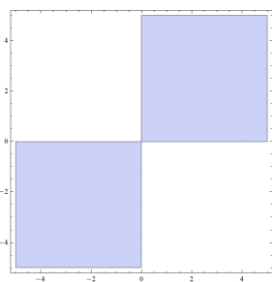
2. Determine and sketch domain of definition of function $f(x, y) = 1 + \arcsin(x + y)$. Describe properties of $D(f)$. Is point $A = [0, 0]$ an interior point of $D(f)$?

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : -\frac{\pi}{2} < x + y < \frac{\pi}{2}\}$, $A \in D(f)$, A is interior point

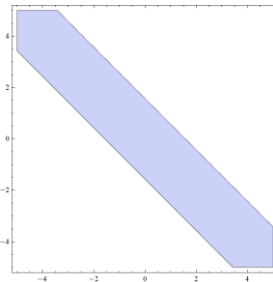
3. Determine and sketch domain of definition of function

$$f(x, y) = \frac{1}{x^2 - y^2 + 1} + \ln(e - y - x^2). \text{ Describe its properties.}$$

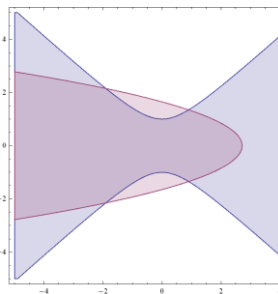
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : x^2 - y^2 + 1 \neq 0 \wedge e - y - x^2 > 0\}$, $D(f)$ is open, not bounded, not simply connected



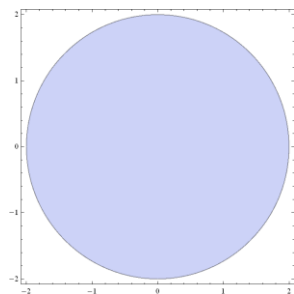
Problem 1



Problem 2



Problem 3



Problem 4

4. Determine and sketch domain of definition of function $f(x, y) = 2 - \sqrt{4 - x^2 - y^2}$.

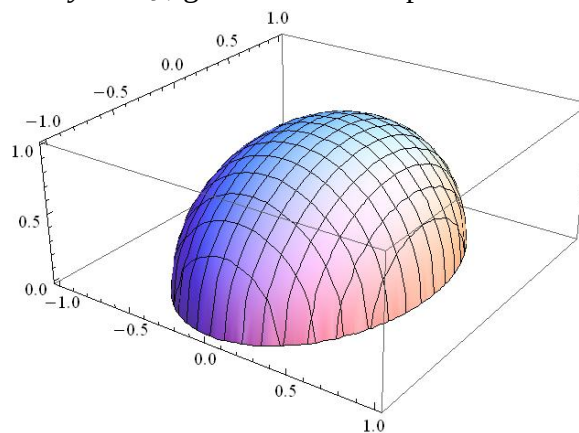
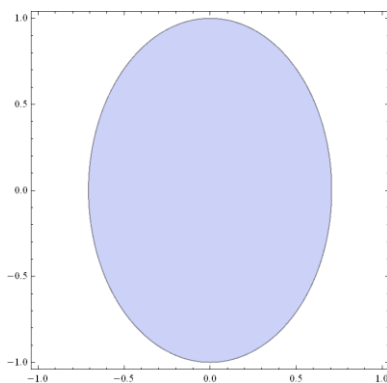
Calculate $f(0, 1)$, $f(-1, 1)$. Describe graph of function f .

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 4 - x^2 - y^2 \geq 0\}$, $f(0, 1) = 2 - \sqrt{3}$, $f(-1, 1) = 2 - \sqrt{2}$, graph is semi-sphere with centre in point $S = [0, 0, 2]$ and radius 2.

5. Determine and sketch domain of definition of function $f(x, y) = \sqrt{1 - 2x^2 - y^2}$.

Sketch and describe graph of function f .

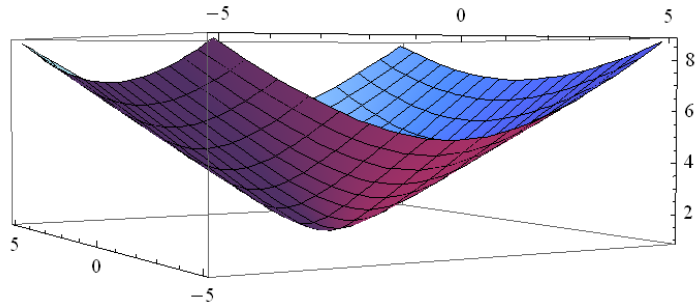
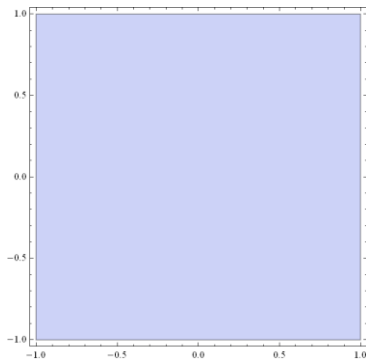
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 1 - 2x^2 - y^2 \geq 0\}$, graf is a semi-ellipsoid



6. Determine and sketch domain of definition of function $f(x, y) = \sqrt{2x^2 + y^2 + 1}$.

Sketch and describe graph of function f .

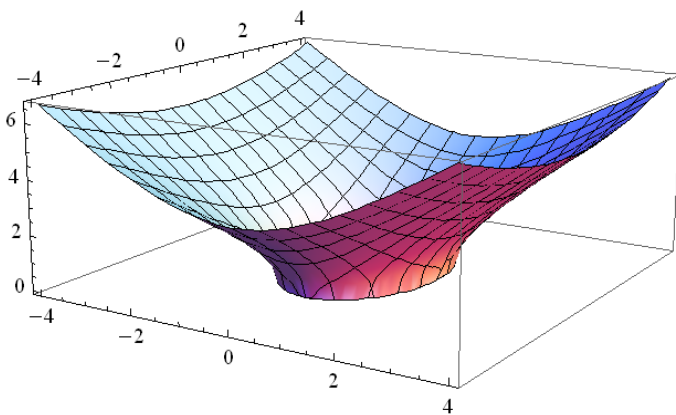
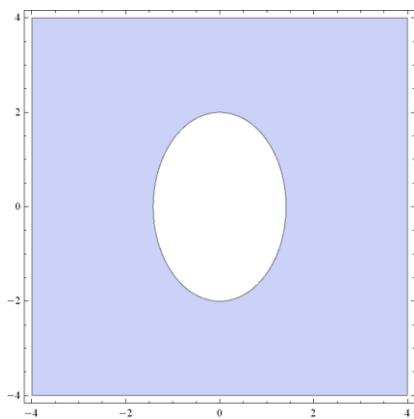
Solution: $D(f) = \{(x, y) \in \mathbb{R}^2 : 2x^2 + y^2 + 1 \geq 0\}$, graph is a half-conical surface



7. Determine and sketch domain of definition of function $f(x, y) = \sqrt{2x^2 + y^2 - 4}$

Sketch and describe graph of function f .

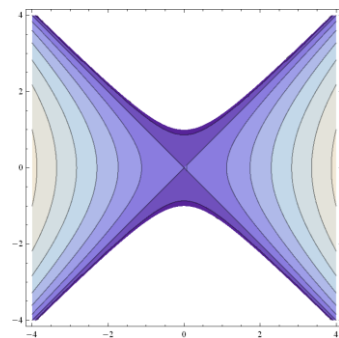
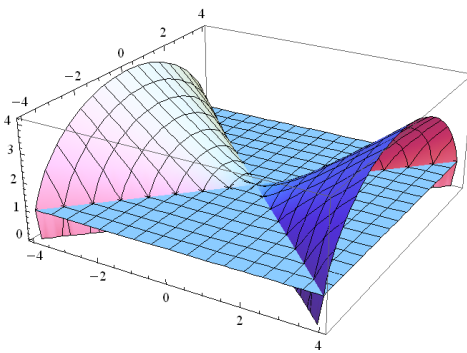
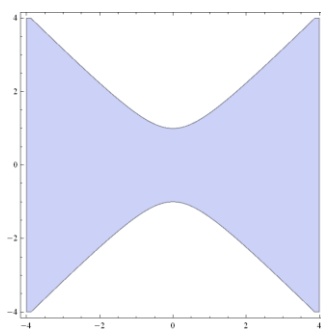
Solution: $D(f) = \{(x, y) \in \mathbb{R}^2 : 2x^2 + y^2 - 4 \geq 0\}$, graph is semi.hyperboloid



8. Determine and sketch domain of definition of function $f(x, y) = \sqrt{x^2 - y^2 + 1}$

Sketch graph of function f , its level lines and find planar intersection by plane $z = 1$.

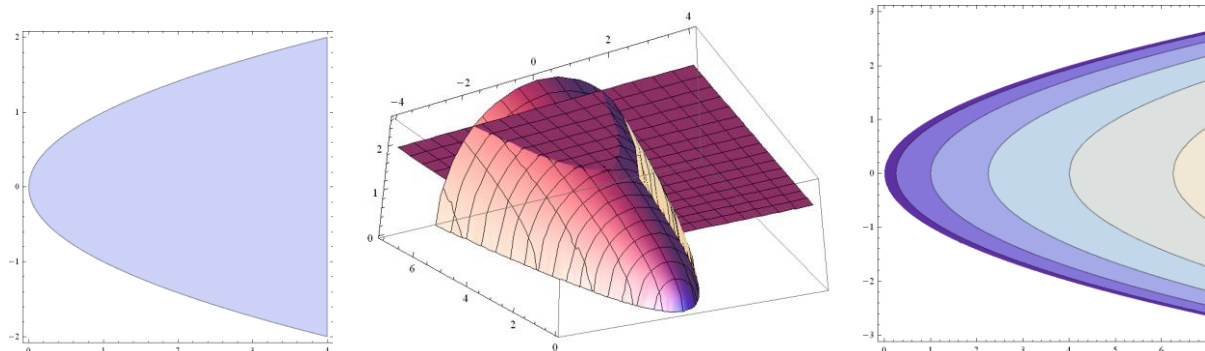
Solution: $D(f) = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 + 1 \geq 0\}$



9. Determine and sketch domain of definition of function $f(x, y) = \sqrt{x - y^2}$.

Sketch graph of function f , its level lines and find planar intersection by plane $z = 2$.

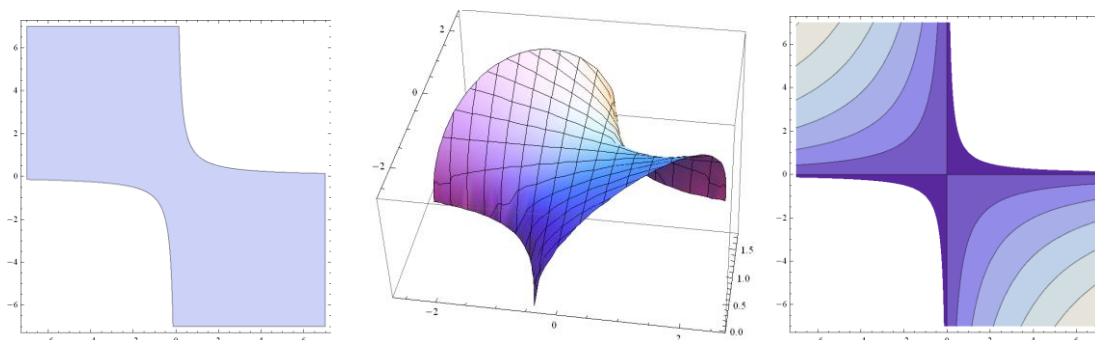
Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : x - y^2 \geq 0\}$



10. Determine and sketch domain of definition of function $f(x, y) = \sqrt{1 - xy}$.

Nakreslite graf danej funkcije. Nakreslite vrstevnice grafu.

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 1 - xy \geq 0\}$



11. Determine and sketch domain of definition of function $f(x, y) = \cos\sqrt{2x^2 + y^2}$.

Sketch graph of function f , its level lines and find planar intersection by plane $z = 0$.

Solution: $D(f) = \{[x, y] \in \mathbb{R}^2 : 2x^2 + y^2 \geq 0\}$

