Analytic-coordinate geometry of the 3-dimensional Euclidean space

1. Write equation of a plane passing through the point $P = [1, 2, 3]$ perpendicularly to the vector: a) $\mathbf{i}$ b) $\mathbf{n} = (3, 2, -5)$ c) $\mathbf{n} = (1, 1, 0)$.
   Solution: a) $x = 1$ b) $3x + 2y - 5z + 8 = 0$ c) $x + y - 3 = 0$

2. Write equations of coordinate planes.
   Solution: Plane xy: $z = 0$, plane xz: $y = 0$, plane yz: $x = 0$

3. Find points in which plane ρ: $x - 3y + 2z - 6 = 0$ intersects coordinate axes.
   Solution: $X = [6, 0, 0]$, $Y = [0, -2, 0]$, $Z = [0, 0, 3]$

4. Show that if in the equation of plane ρ: $ax + by + cz + d = 0$ holds
   a) $d = 0$, then plane ρ is passing through the origin
   b) $a = b = 0$, then plane ρ is parallel to the coordinate plane xy
   c) $c = 0$, then plane ρ is perpendicular to coordinate plane xy.
   Solution: a) $ax + by + cz = 0$, $a.0 + b.0 + c.0 = 0$
   b) $cz + d = 0$, z-coordinates of all points in the plane are constant, $z = -d/c$
   c) $ax + by + d = 0$, plane normal vector $\mathbf{n} = (a, b, 0) = a.\mathbf{i} + b.\,\mathbf{j}$, and $\mathbf{n} \,.\, \mathbf{k} = 0$

5. Write equations of a plane determined by:
   a) points $A = [0, 2, 2]$, $B = [-4, 0, 3]$, $C = [3, 1, 0]$
   b) points $A = [1, 0, 0]$, $B = [0, 0, 1]$, and perpendicular to the coordinate plane xz
   c) point $M = [1, -1, 2]$, and parallel to the coordinate plane xz.
   Solution: a) $x - y + 2z - 2 = 0$; $x = -4t + 3s$, $y = 2 - 2t - s$, $z = 2 + t - 2s$, $t, s \in \boldsymbol{R}$
   b) $x + z = 1$; $x = 1 + t$, $y = s$, $z = -t$, $t, s \in \boldsymbol{R}$
   c) $y + 1 = 0$; $x = 1 + t$, $y = -1$, $z = 2 + s$, $t, s \in \boldsymbol{R}$

6. Write parametric equations of a line l
   a) passing through the point $P = [-1, 0, 3]$, and parallel to the coordinate axis y
   b) determined by points $A = [-1, 0, 3]$, $B = [2, 1, -1]$.
   Solution: a) $x = -1$, $y = t$, $z = 3$, $t \in \boldsymbol{R}$ b) $x = -1 + 3t$, $y = t$, $z = 3 - 4t$, $t \in \boldsymbol{R}$

7. Write parametric equation of the coordinate axes.
   Solution: axis x: $x = t$, $y = 0$, $z = 0$, $t \in \boldsymbol{R}$
   axis y: $x = 0$, $y = 0$, $z = 0$, $t \in \boldsymbol{R}$
   axis z: $x = 0$, $y = 0$, $z = t$, $t \in \boldsymbol{R}$

8. Write parametric equations of a line determined by the general equations
   $x - y + z - 5 = 0$, $x + 2y - 7 = 0$.
   Solution: $x = 7 - 2t$, $y = t$, $z = -2 + 3t$, $t \in \boldsymbol{R}$

9. Write general equations of a line determined by the following parametric equations $x = 2 - t$, $y = 1 + 4t$, $z = -3 + 2t$, $t \in \boldsymbol{R}$.
Solution: $2x + z - 1 = 0$, $y - 2z - 7 = 0$

10. Determine mutual position of lines:
a) l_1: $x = -3 + 2t$, $y = 4 - t$, $z = 1 + 3t$, l_2: $x = -6 + t$, $y = 5 - t$, $z = 1 + 6t$, $t \in \boldsymbol{R}$
b) l_1: $x = -1 + t$, $y = 18 + 9t$, $z = 10 + 5t$, $t \in \boldsymbol{R}$, l_2: $x + y - 2z + 3 = 0$, $3x - 2y + 3z + 9 = 0$
Solution: a) lines are intersecting in the point $P = [-7, 6, -5]$
b) lines coincide.

11. Find parametric equation of line l passing through the point $M = [2, 3, -1]$ perpendicularly to the plane ρ: $x + 2y - z = 0$.
Solution: $x = 2 + t$, $y = 3 + 2t$, $z = -1 - t$, $t \in \boldsymbol{R}$

12. Find mutual position of line l: $x = 2 - t$, $y = 1 + 3t$, $z = -3 + 2t$, $t \in \boldsymbol{R}$ and plane ρ: $x - 2y + 3z - 1 = 0$.
Solution: Line and plane are intersecting in the point $P = [12, -29, -23]$.

13. Find equation of an orthogonal projection of the line l: $x = -3 + 2t$, $y = 4 + t$, $z = -1 - 3t$, $t \in \boldsymbol{R}$ onto the plane ρ: $x - y + 2z - 4 = 0$.
Solution: $x + 7y + 3z - 22 = 0$, $x - y + 2z - 4 = 0$

14. Find coordinates of point M' symmetric to the point $M = [5, 2, -6]$ with respect to the. plane ρ: $x - y - 4z - 9 = 0$.
Solution: $M' = [3, 4, 2]$

15. Find equation of a line k passing through the point $M = [2, 3, 2]$, perpendicular to the line l: $x = -1 - t$, $y = 2t$, $z = 3 + 2t$, $t \in \boldsymbol{R}$, and intersecting it.
Solution: $x = 2 + 28t$, $y = 3 + 25t$, $z = 2 - 11t$, $t \in \boldsymbol{R}$