

Vectors and operations with vectors

1. If $\mathbf{a} = (2, 1, 3)$ and $\mathbf{b} = (1, -2, 0)$ are two vectors, calculate the following $\mathbf{a} + \mathbf{b}$, $|\mathbf{a}|$, $|\mathbf{b}|$, $\mathbf{a} \cdot \mathbf{b}$, $3\mathbf{a} - \mathbf{b}$, $\mathbf{a} \times \mathbf{b}$.

Solution: $\mathbf{a} + \mathbf{b} = (3, -1, 3)$, $|\mathbf{a}| = \sqrt{14}$, $|\mathbf{b}| = \sqrt{5}$, $\mathbf{a} \cdot \mathbf{b} = 0$ ($\mathbf{a} \perp \mathbf{b}$),

$$3\mathbf{a} - \mathbf{b} = (5, 5, 9), \quad \mathbf{a} \times \mathbf{b} = (6, 3, -5).$$

2. Find out if vectors $\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$, $\mathbf{c} = 2\mathbf{k}$ are linearly independent.

Solution: Vectors are linearly independent, $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = 2$.

3. Find coordinates of vector $\mathbf{w}$ perpendicular to vectors $\mathbf{a} = (1, 0, 1)$ and $\mathbf{b} = (-1, 1, 0)$, and calculate its length.

Solution: $\mathbf{w} = (-1, -1, 1)$, $|\mathbf{w}| = \sqrt{3}$.

4. Calculate area of a parallelogram determined by vectors $\mathbf{a} = (2, 1, 2)$, $\mathbf{b} = (0, 4, 0)$.

Solution: $P = 8\sqrt{2}$.

5. Find angle formed by vectors: a) $\mathbf{a} = (2, 0, 0)$, $\mathbf{b} = (0, 3, 4)$ b) $\mathbf{a} = (1, 1, 1)$, $\mathbf{b} = (0, 0, 3)$
c) $\mathbf{a} = (1, 1, 0)$, $\mathbf{b} = (0, 1, 1)$ d) $\mathbf{a} = (2, 0, 2)$, $\mathbf{b} = (1, 1, 0)$

Solution: a) $\varphi = \frac{\pi}{2}$ b) $\varphi = 0,96\text{rd}$ c) $\varphi = \frac{\pi}{4}$ d) $\varphi = \frac{\pi}{3}$

6. Calculate volume of parallelepiped determined by vectors $\mathbf{a} = (2, 4, 0)$, $\mathbf{b} = (1, 1, 5)$, $\mathbf{c} = (-2, 3, 2)$.

Solution: $V = 74$

7. Are vectors $\mathbf{a} = (-1, 2, 4)$, $\mathbf{b} = (3, 0, 2)$, $\mathbf{c} = (-2, 1, 4)$ coplanar?

Solution: Vectors are not coplanar, $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = -18$

8. Find coordinates of vector $\mathbf{w}$ that is perpendicular to vector $\mathbf{a} = (2, 3, 4)$ and collinear with vector $\mathbf{b} = (1, -2, ?)$, while its length is $\sqrt{150}$.

Solution: $\mathbf{w} = (5, -10, 5)$

9. Evaluate area of a parallelogram determined by vectors $\mathbf{a} = (2, 0, 2)$, $\mathbf{b} = (2, 2, 0)$ using the formula that includes the angle φ formed by the two vectors.

Solution: $P = |\mathbf{a}| \cdot |\mathbf{b}| \cdot \sin \varphi = 4\sqrt{3}$

10. Determine the position of vectors $\mathbf{a} = (7, -2, 1)$, $\mathbf{b} = (2, 5, -4)$, $\mathbf{c} = (8, -19, 14)$.

Solution: $\mathbf{a} \perp \mathbf{b}$, $\mathbf{c} = 2\mathbf{a} - 3\mathbf{b}$