Evaluation of double integral of function $f(x, y)$ on interval in 2D

Integrate[f[x,y], {x, ∗ , ∗}, {y, ∗ , ∗}]

Evaluation of triple integral of funtion $f(x, y, z)$ on interval in 3D

Integrate[f[x,y,z], {x, ∗ , ∗}, {y, ∗ , ∗}, {z, ∗ , ∗}]

Evaluation of double integral of function $f(x, y)$ on regular region M with respect to x

$M = \{[x, y] \in E_2 : a \le x \le b, g(x) \le y \le h(x)\}$

Integrate[f[x,y], {x, a, b}, {y, g(x), h(x)}]

Evaluation of double integral of function $f(x, y)$ on regular region M with respect to y

$M = \{[x, y] \in E_2 : g(y) \le x \le h(y), c \le y \le d\}$

Integrate[f[x,y], {y, c, d}, {x, g(y), h(y)}]

Plot of a region with boundary curves determined by equations eq1, eq2

ContourPlot[{eq1, eq2}, {x, ∗ , ∗}, {y, ∗ , ∗}]

Common points on graphs of functions $g(x)$, $h(x)$

r=Solve[g[x] ==h[x], x]
r=Reduce[g[x] ==h[x], x]
a=x/.r[[1]]
b=x/.r[[2]]
r=FindRoot[g[x] ==h[x], {x,x0}]

RegionPlot[{g[x]≤y≤h[x]}, {x, a, b}, {y, ∗ , ∗}]

Evaluation of triple integral of function $f(x, y, z)$ on regular region T with respect to xy

$T = \{[x, y, z] \in E_3 : a \le x \le b, g(x) \le y \le h(x), z_1(x, y) \le z \le z_2(x, y)]$

Integrate[f[x,y,z], {x, a, b}, {y, g(x), h(x)}, {z, z1(x,y), z1(x,y)}]

RegionPlot3D[{g[x]≤y≤h[x], z1[x,y]≤z≤z2[x,y]}, {x, a, b}, {y, ∗ , ∗},{z, ∗ , ∗}]

Problems

1. Evaluate double integrals:

a) $f(x, y) = \dfrac{x^2}{1+y^2}, I = \langle 0,1\rangle \times \langle 0,1\rangle$

b) $f(x, y) = ye^{x+y}, I = \langle 0,1\rangle \times \langle 0,1\rangle$

c) $f(x, y) = x^2 + y^2, I = \langle -1,2\rangle \times \langle 0,1\rangle$

d) $f(x, y) = x \sin y, I = \langle 2,4\rangle \times \left\langle 0, \dfrac{\pi}{2}\right\rangle$

e) $f(x, y) = x + y,\ x \ge 0, y \ge 0, x^2 + y^2 \le 1$

f) $f(x, y) = 7xy, 1 - x \le y \le 1 - x^2$

g) $f(x, y) = xy$, on M bounded by graphs of functions $y = 1, y = \sqrt{x}, y = -x$

h) $f(x, y) = e^x y^2$, on M bounded by curves $x = y^2, y = e^x - 2$

2. Evaluate triple integrals:

a) $f(x, y, z) = \sin x \sin y \cos z, I = \langle 0, \pi\rangle \times \langle 0, \pi\rangle \times \left\langle 0, \dfrac{\pi}{2}\right\rangle$

b) $f(x, y, z) = \dfrac{y}{x+z}, I = \langle 0,1\rangle^3$

c) $f(x, y, z) = x + y + z, -\sqrt{1-y^2-z^2} \le x \le \sqrt{1-y^2-z^2}, -\sqrt{1-z^2} \le y \le \sqrt{1-z^2}, -1 \le z \le 1$

d) $f(x, y, z) = 2x^2 - 4y^3, (y, z) \in \langle 0,1\rangle^2, 0 \le x \le 3y$

e) $f(x, y, z) = y \cos(x + z)$, on M defined by relations $y \ge 0, x = \pi, y = \sqrt{x}, -\pi \le z \le \pi$

f) $f(x, y, z) = xyz$, on M defined by relations $0 \le x \le 1, 0 \le y \le 1 - x, 1 \le z \le 2 - x - y$