Partial derivatives of function $f(x, y)$
D[f[x,y], x], D[f[x,y], y], D[f[x,y], x,y], D[f[x,y], x,x], D[f[x,y], y,y]

or **D[f[x,y], {x, n}], D[f[x,y], {y, n}]**

Local extrema of function $f(x, y)$
f[x_, y_] = formula
fx = D[f[x,y], x]
fy = D[f[x,y], y]
d1[x_,y_] = D[f[x,y], x,x]
d2 = {{D[f[x,y], x,x], D[f[x,y], x,y]},{ D[f[x,y], y,x], D[f[x,y], y,y]}};MatrixForm[d]
He[x_,y_] = Det[d2]

Stationary points of function $f(x, y)$
Solve[{fx == 0, fy == 0}, {x, y}]
Graph of function $f(x, y)$ in the neighbourhood of stationary points

ContourPlot[f[x, y], {x, ∗ , ∗}, {y, ∗ , ∗}]

gf = Plot3D[f[x, y], {x, ∗ , ∗}, {y, ∗ , ∗}]

Plot of stationary points

gb = ListPointPlot3D[{{∗ , ∗, f[∗ , ∗]}, {∗ , ∗, f[∗ , ∗]}, ...}, PlotStyle->PointSize[0.02]]
Show[gf, gb}]

Constrained extrema of function $f(x, y)$
Constraint **g[x_] = formula**
Special partial function **F[x_] = f[x, g[x]]**

Plot of set V determined by constraint (in black) and graph of partial function $F(x)$ (in red)

gk = ParametricPlot3D[{{x, g[x], 0]}, {x , g[x], F[x]}}, {x, ∗ , ∗},
PlotStyle → {Thick, {RGBColor[1, 0, 0], Thick}}]

Plot of extremal points

min (max) = ParametricPlot3D[{∗, g[∗], t]}, {t, 0 , F[∗]},
PlotStyle → {{RGBColor[0, 1, 0], Thick}}]

Problems

1. Evaluate partial derivatives of function f in point A:

   a) $f(x,y)=\cos\sqrt{2x^2+y^2}, A=[1,1]$ b) $f(x,y)=1+\arcsin(x+y), A=[0,0]$

   c) $f(x,y)=\dfrac{1}{x^2-y^2+1}, A=[2,7]$ d) $f(x,y)=\ln(e-y-x^2), A=[e,e]$

   e) $f(x,y)=Arctg\dfrac{x}{y}, A=[1,2]$ f) $f(x,y)=e^{xy}, A=[3,2]$

2. Find all local extrema of function f and plot its graph in their neighbourhood:D

   a) $f(x,y)=x^2+y^3-2x-3y$ b) $f(x,y)=1-\sqrt[3]{x^2+y^2}$

   c) $f(x,y)=e^{-x^2-y^2}(x^2+2y^2)$ d) $f(x,y)=2x^2+3y^2-8\ln(xy)$

   e) $f(x,y)=x^3+xy^2-27x$ f) $f(x,y)=x^2+y^3-2x-3y$

3. Find constrained extrema of function $f(x, y)$ with constraint $g(x, y) = 0$ and plot graphs:

a) $f(x,y) = 1 + x^2 + y^2, g(x,y) = xy - 1$ b) $f(x,y) = 1 - x^2 - y^2, g(x,y) = x + y - 1$

c) $f(x,y) = x^3 + y^3, g(x,y) = x + y - 3$ d) $f(x,y) = x^2 - y^2, g(x,y) = x^2 + y^2 - 4$

e) $f(x,y) = 10 + y - \frac{x^3}{3} + \ln(x - y), g(x,y) = x - y - 1$

4. Find gobal extrema of function $f(x, y)$ on the set M:

a) $f(x,y) = 10 + y - \frac{x^3}{3} + \ln(x - y), M = \{[x,y] \in E_2 : y = x - 1, -1 \le x \le 3\}$

b) $f(x,y) = x^2 - y^2, M = \{[x,y] \in E_2 : x^2 + y^2 \le 4\}$

c) $f(x,y) = x^2 + 2y + 2, M = \{[x,y] \in E_2 : x = \sqrt{4 - y^2}, -1 \le y \le 2\}$

d) $f(x,y) = x^2 - 2y^2 + 4xy - 6x + 21, M = \{[x,y] \in E_2 : x \ge 0, y \ge 0, y \le 3 - x\}$

e) $f(x,y) = \cos^2 x + \sin^2 y, M = \left\{[x,y] \in E_2 : -\frac{\pi}{2} \le x \le \frac{\pi}{2}, -\frac{\pi}{2} \le y \le \frac{\pi}{2}\right\}$