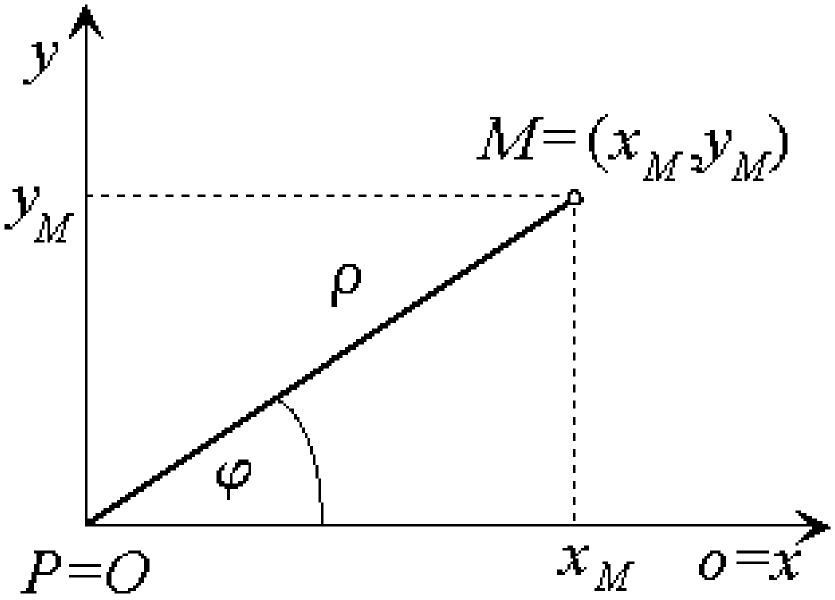
Transformations of multiple integrals

Polar coordinate system



$\boldsymbol{x = \rho \cos \varphi}$

$\boldsymbol{y = \rho \sin \varphi}$

$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = arctg\frac{y}{x}, x \neq 0,$$

$$\begin{cases} \varphi = \dfrac{\pi}{2}, x = 0 \wedge y > 0 \\ \varphi = \dfrac{3\pi}{2}, x = 0 \wedge y < 0 \end{cases}$$

$$M^* = \{[\rho, \varphi] \in E_2\} \rightarrow M = \{[x, y] \in E_2\}$$

$$J(\rho, \varphi) = \begin{vmatrix} \cos\varphi & -\rho\sin\varphi \\ \sin\varphi & \rho\cos\varphi \end{vmatrix} = \rho$$

$$\iint\limits_{M} f(x,y)\,dx\,dy = \iint\limits_{M^*} f(\rho\cos\varphi, \rho\sin\varphi)\rho\, d\rho\, d\varphi$$

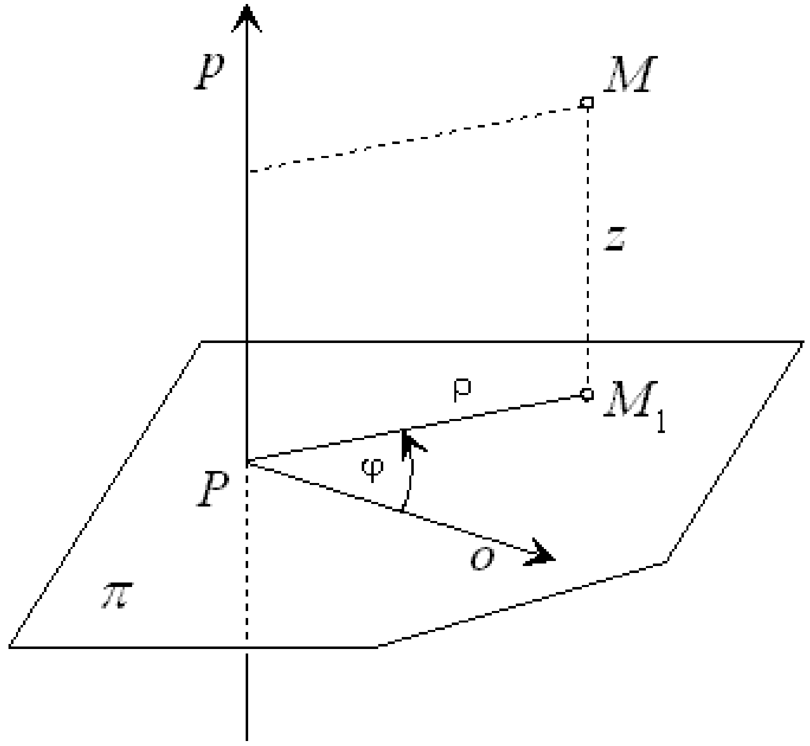
$$M^* = \{[\rho,\varphi] \in E_2, 0 \le \rho < \infty, 0 \le \varphi \le 2\pi\}$$

Cylindrical coordinate system

x* = *ρ* cos *φ

y* = *ρ* sin *φ

z = z



$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = \arccos \frac{x}{\sqrt{x^2 + y^2}}$$

$$J(\rho, \varphi, z) = \begin{vmatrix} \cos\varphi & -\rho\sin\varphi & 0 \\ \sin\varphi & \rho\cos\varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} = \rho$$

$$T^* = \{[\rho, \varphi, z] \in E_3, 0 \le \rho < \infty, 0 \le \varphi \le 2\pi, -\infty < z < \infty\}$$

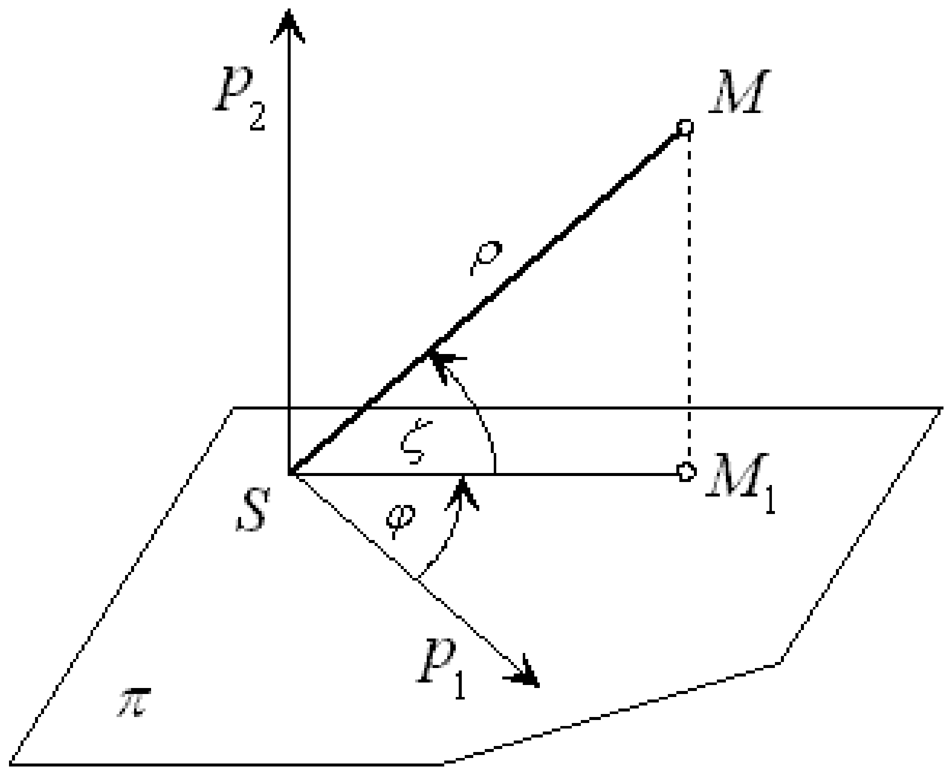
$$\iiint_T f(x, y, z)\,dx\,dy\,dz = \iiint_{T^*} f(\rho\cos\varphi, \rho\sin\varphi, z)\,\rho\,d\rho\,d\varphi\,dz$$

Spherical coordinate system

$$x = \rho \cos \varphi \ \cos \zeta$$

$$y = \rho \sin \varphi \cos \zeta$$

$$z = \rho \ \sin \zeta$$



$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$\varphi = \arccos \frac{x}{\sqrt{x^2 + y^2}}$$

$$\zeta = \arcsin \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$J(\rho,\varphi,z)=\begin{vmatrix}\cos\varphi\cos\zeta & -\rho\sin\varphi\cos\zeta & -\rho\cos\varphi\sin\zeta\\ \sin\varphi\cos\zeta & \rho\cos\varphi\cos\zeta & -\rho\sin\varphi\sin\zeta\\ \sin\zeta & 0 & \rho\cos\zeta\end{vmatrix}=\rho^2\cos\zeta$$

$$T^*=\left\{[\rho,\varphi,\zeta]\in E_3, 0\le\rho<\infty, 0\le\varphi\le 2\pi, -\frac{\pi}{2}<\zeta<\frac{\pi}{2}\right\}$$

$$\iiint_T f(x,y,z)\,dx\,dy\,dz=$$

$$=\iiint_{T^*} f(\rho\cos\varphi\cos\zeta,\rho\sin\varphi\cos\zeta,\rho\sin\zeta)\rho^2\cos\zeta\,d\rho\,d\varphi\,d\zeta$$