

Substitúcie množných integrálov

Polárna súradnicová sústava

$$x = \rho \cos \varphi$$

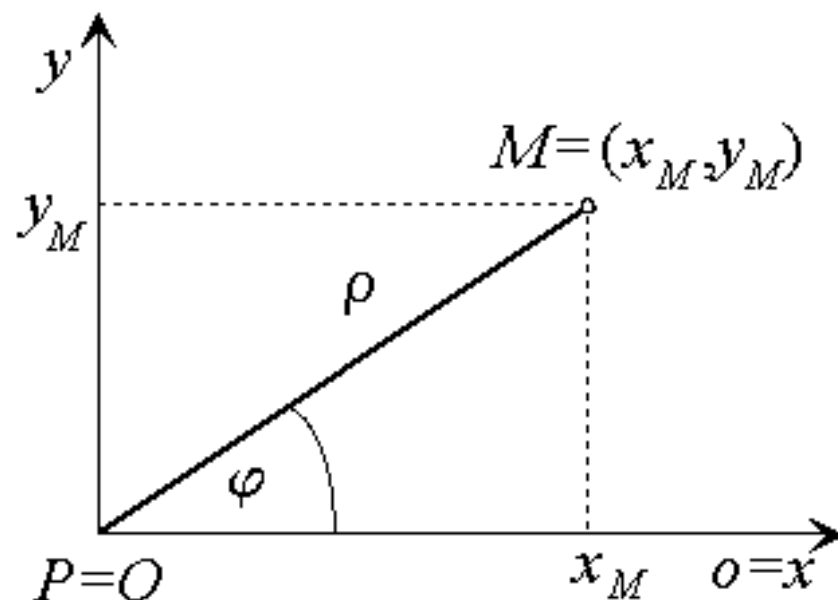
$$y = \rho \sin \varphi$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = \operatorname{arctg} \frac{y}{x}$$

$$J(\rho, \varphi) = \begin{vmatrix} \cos \varphi & -\rho \sin \varphi \\ \sin \varphi & \rho \cos \varphi \end{vmatrix} = \rho$$

$$\iint_M f(x, y) dx dy = \iint_{M^*} f(\rho \cos \varphi, \rho \sin \varphi) \rho d\rho d\varphi$$

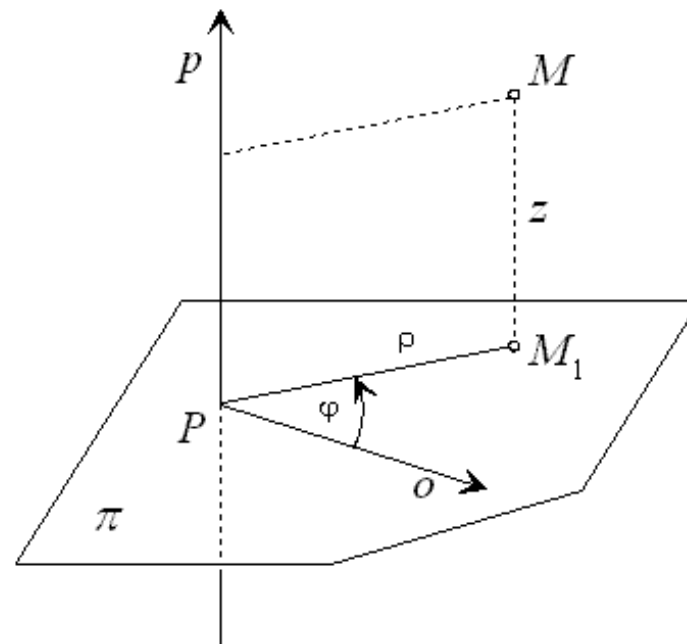


Cylindrická súradnicová sústava

$$x = \rho \cos \varphi$$

$$y = \rho \sin \varphi$$

$$z = z$$



$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = \arccos \frac{x}{\sqrt{x^2 + y^2}}$$

$$J(\rho, \varphi, z) = \begin{vmatrix} \cos \varphi & -\rho \sin \varphi & 0 \\ \sin \varphi & \rho \cos \varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} = \rho$$

$$\iiint_T f(x, y, z) dx dy dz = \iiint_{T^*} f(\rho \cos \varphi, \rho \sin \varphi, z) \rho d\rho d\varphi dz$$

Sférická súradnicová sústava

$$x = \rho \cos \varphi \cos \zeta$$

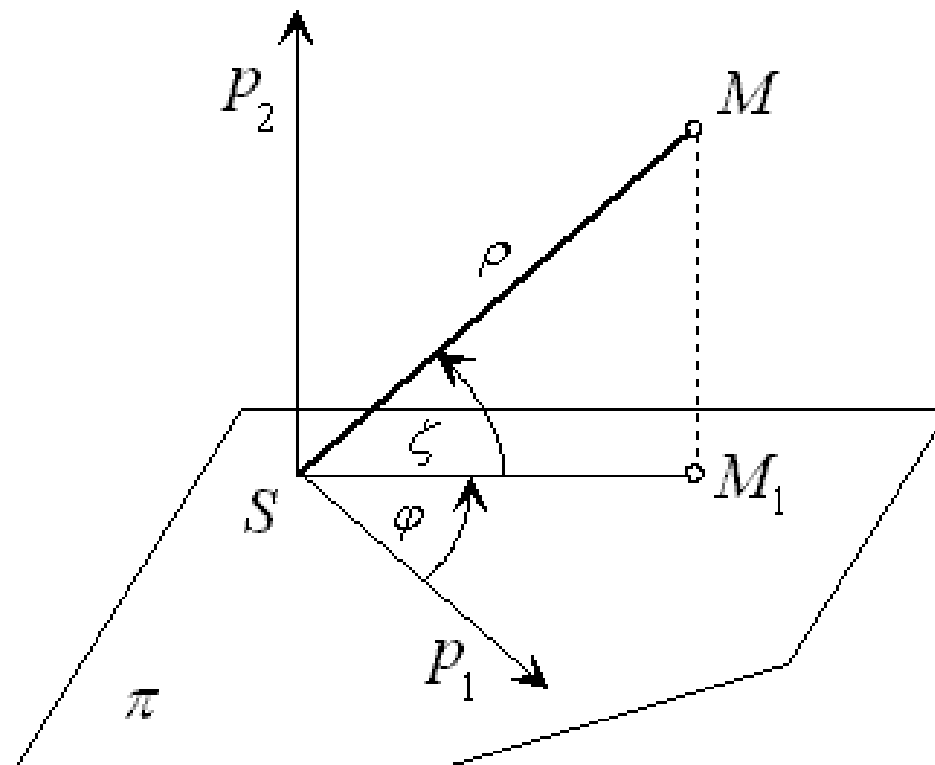
$$y = \rho \sin \varphi \cos \zeta$$

$$z = \rho \sin \zeta$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$\varphi = \arccos \frac{x}{\sqrt{x^2 + y^2}}$$

$$\zeta = \arcsin \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$



$$J(\rho, \varphi, z) = \begin{vmatrix} \cos \varphi \cos \zeta & -\rho \sin \varphi \cos \zeta & -\rho \cos \varphi \sin \zeta \\ \sin \varphi \cos \zeta & \rho \cos \varphi \cos \zeta & -\rho \sin \varphi \sin \zeta \\ \sin \zeta & 0 & \rho \cos \zeta \end{vmatrix} = \rho^2 \cos \zeta$$

$$\iiint_T f(x, y, z) dx dy dz =$$

$$= \iiint_{T^*} f(\rho \cos \varphi \cos \zeta, \rho \sin \varphi \cos \zeta, \rho \sin \zeta) \rho^2 \cos \zeta d\rho d\varphi d\zeta$$