

Aplikácie množných integrálov

Objem telesa v E_3

$$T = \{[x, y, z] \in E_3 : [x, y] \in M, 0 \leq z \leq f(x, y)\}$$

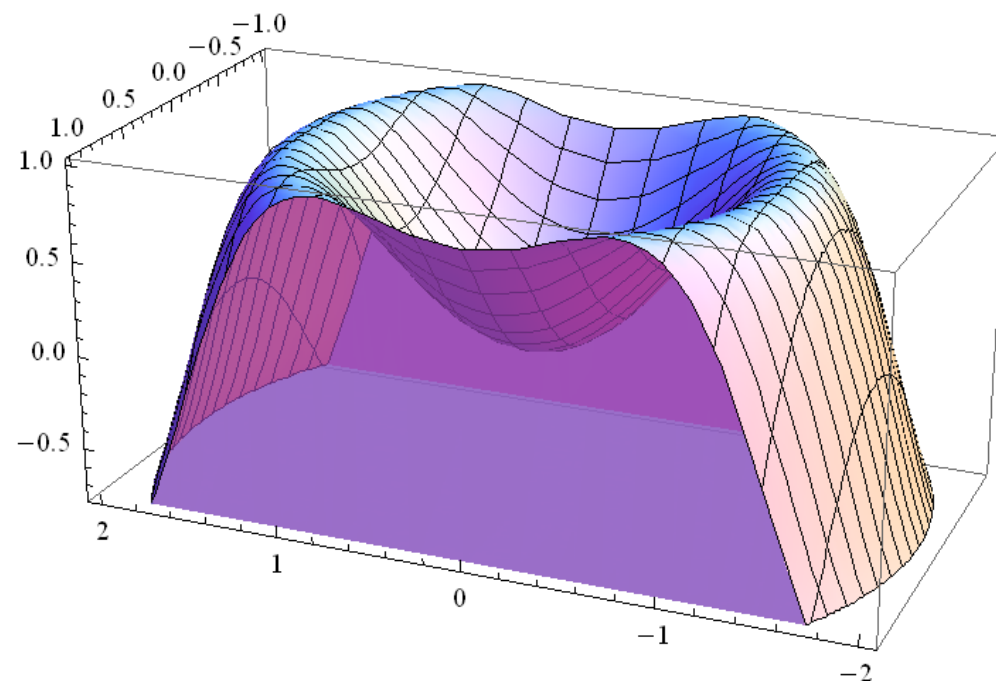
$$M = \{[x, y] \in E_2 : a \leq x \leq b, y_1(x) \leq y \leq y_2(x)\}$$

Spojité funkcia $f(x, y) \geq 0$ na M

Objem telesa

$$V(T) = \iint_M f(x, y) dx dy$$

$$V(T) = \iiint_T 1 dx dy dz$$



Objem telesa v E_3

$$T = \{[x, y, z] \in E_3 : [x, y] \in M, g(x, y) \leq z \leq f(x, y)\}$$

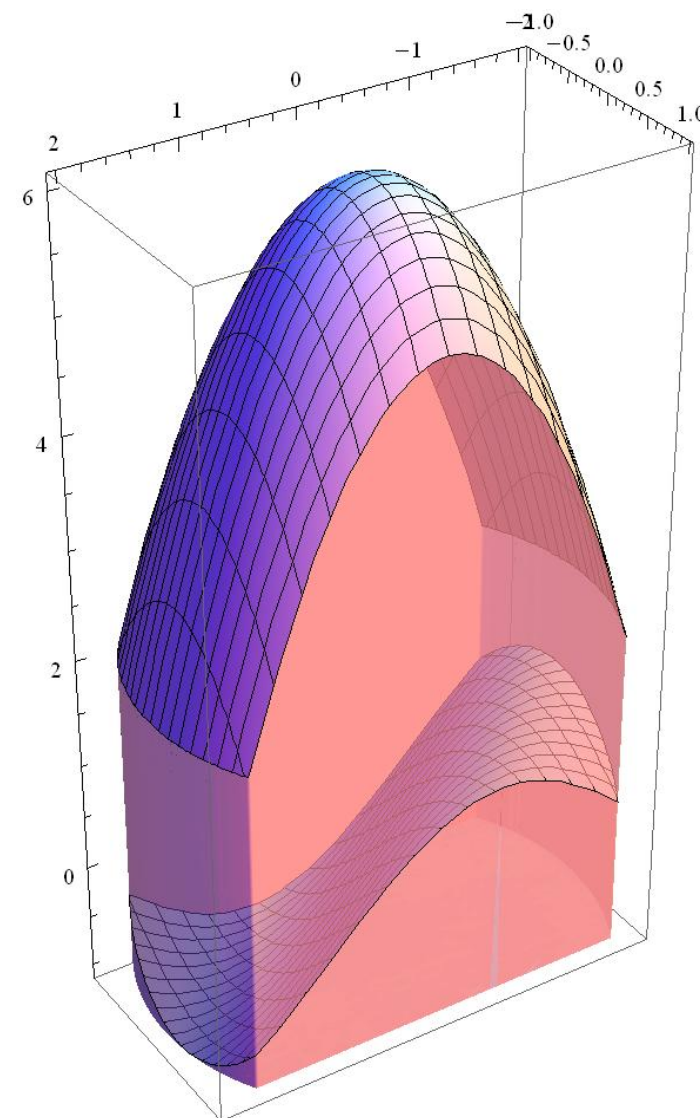
$$M = \{[x, y] \in E_2 : a \leq x \leq b, y_1(x) \leq y \leq y_2(x)\}$$

Spojité funkcia $f(x, y) \geq 0$ na M

Objem telesa

$$V(T) = \iint_M (f(x, y) - g(x, y)) dx dy$$

$$V(T) = \iiint_T 1 dx dy dz$$

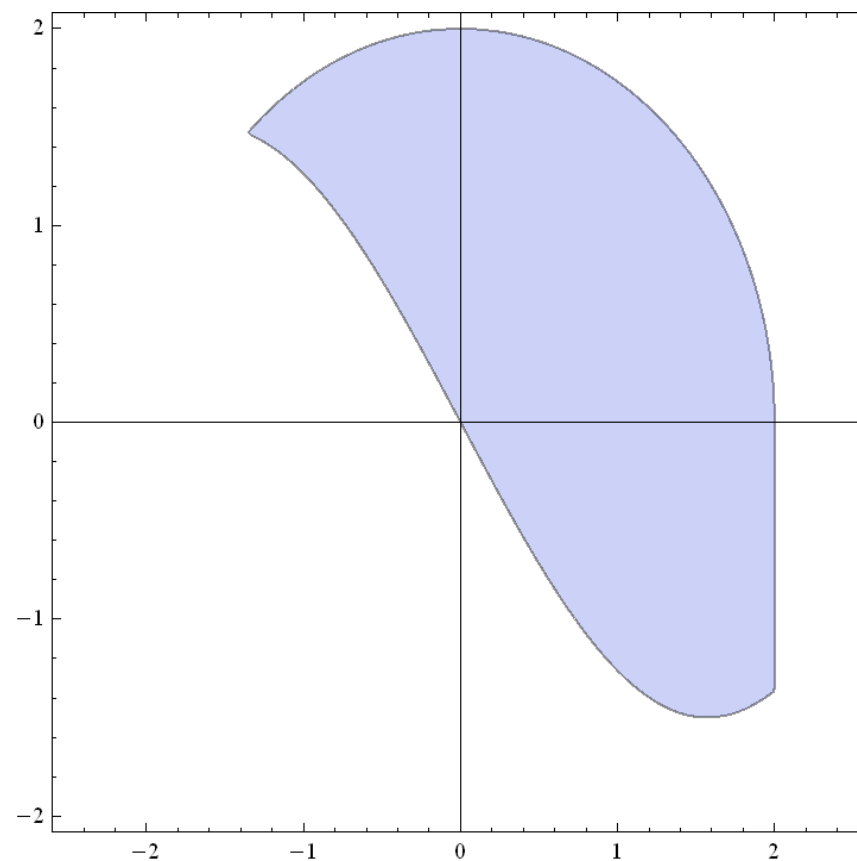


Obsah elementárnej oblasti

$$M = \{[x, y] \in E_2 : a \leq x \leq b, f_1(x) \leq y \leq f_2(x)\}$$

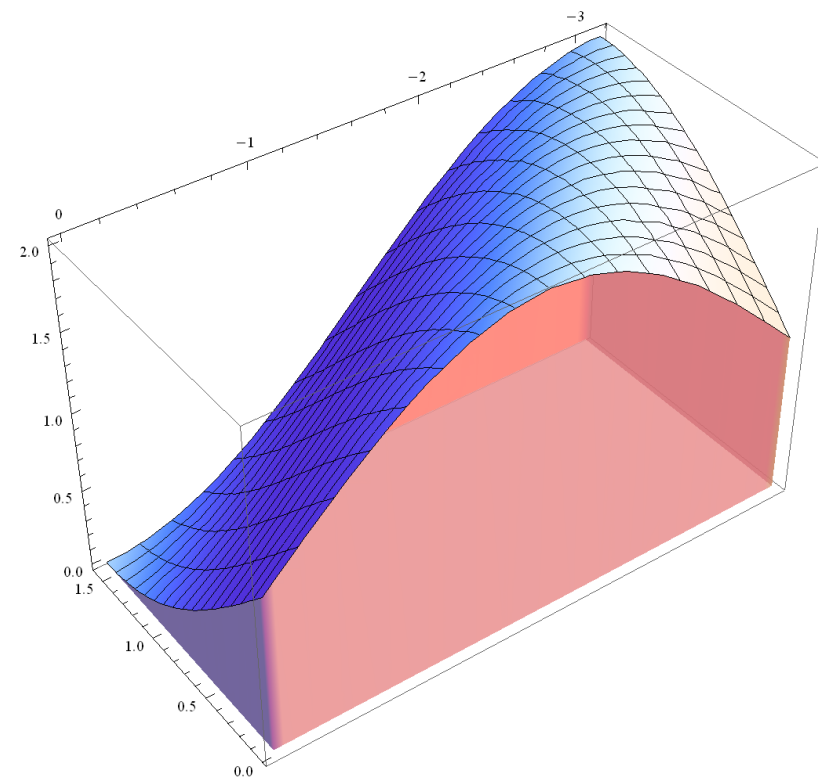
$$M' = \{[x, y] \in E_2 : g_1(y) \leq x \leq g_2(y), c \leq y \leq d\}$$

$$P(M) = \iint_M 1 dx dy$$



Obsah časti grafu spojitej funkcie

$$\sigma = \{[x, y, z] \in \mathbb{E}_3 : [x, y] \in M, z = f(x, y)\}$$



$$S(\sigma) = \iint_M \sqrt{1 + (f'_x(x, y))^2 + (f'_y(x, y))^2} dx dy$$

$M \subset E_2$ je hmotná elementárna oblasť, ktorej hustota je v každom bode $[x, y] \in M$ určená hodnotou spojitaj nezápornej funkcie $\mu(x, y)$

Hmotnosť množiny M

$$m = \iint_M \mu(x, y) dx dy$$

Súradnice ťažiska $T = [x_T, y_T]$ množiny M

$$x_T = \frac{1}{m} \iint_M x \cdot \mu(x, y) dx dy \quad y_T = \frac{1}{m} \iint_M y \cdot \mu(x, y) dx dy$$

$M \subset E_3$ je hmotná elementárna oblasť, ktorej hustota je v každom bode $[x, y, z] \in M$ daná hodnotou spojitej nezápornej funkcie $\mu(x, y, z)$

Hmotnosť množiny M

$$m = \iiint_M \mu(x, y, z) dx dy dz$$

Súradnice ťažiska $T = [x_T, y_T, z_T]$ množiny M

$$x_T = \frac{1}{m} \iiint_M x \cdot \mu(x, y, z) dx dy dz \quad y_T = \frac{1}{m} \iiint_M y \cdot \mu(x, y, z) dx dy dz$$

$$z_T = \frac{1}{m} \iiint_M z \cdot \mu(x, y, z) dx dy dz$$