PROBLEMS ON VECTOR FUNCTIONS I

Problem 1: Find vector function whose graph is a line segment AB, $A = [-1,2,1]$, $B = [1,0,-3]$.
Solution: $\boldsymbol{r}(t) = (-1+2t,\ 2-2t,\ 1-4t)$, $t \in \langle 0, 1\rangle$

Problem 2: Find vector function defined for $t \in \langle 0, 1\rangle$, whose scalar coordinate functions are polynomials and whose graph is passing through points $A = [2, 2,1]$, $B = [1, 3, 4]$, $C = [0,4, 2]$.
Solution: $\boldsymbol{r}(t) = (2 - 2t,\ 2 + 2t,\ 1 + 11t - 10t^2)$, $t \in \langle 0, 1\rangle$.

Problem 3: Determine domain of definition and range of vector function
$\boldsymbol{f}(t) = (-1 + 2t^3,\ t^2+2t+4, 3t)$, find its value in points $t = 0$, $t = 1$, $t = -1$ and sketch these vectors.
Solution: $D(\boldsymbol{f}) = \boldsymbol{R}$, $H(\boldsymbol{f}) \subset \boldsymbol{V}^3(\boldsymbol{R})$, set of vectors, including vectors
$\boldsymbol{f}(0) = (-1, 4, 0)$, $\boldsymbol{f}(1) = (1, 7, 3)$, $\boldsymbol{f}(-1) = (-3, 3, -3)$.

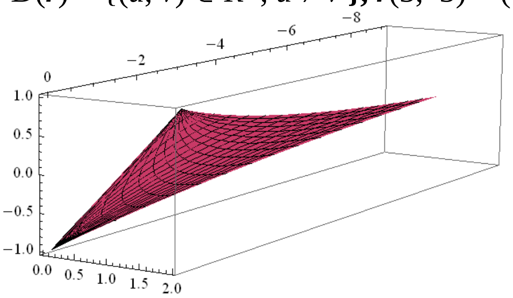
Problem 4: Determine domain of definition and range of vector function
$\mathbf{r}(t) = \left(\dfrac{a}{\cos t}, b\tan t\right), a, b \in R$, and sketch its graph.
Solution: $t \in \langle 0, 2\pi\rangle - \{\pi/2, 3\pi/2\}$**,** range of values is set of vectors forming with positive semi-axis x angles in intervals $(-\varphi, \varphi) \cup (\pi - \varphi, \pi + \varphi)$, $\varphi = \operatorname{atan}(b/a)$, graph is hyperbola.

Problem 5: Find such value of variable t, for which the size of vector, which is the value of vector function $\boldsymbol{r}(t) = (2 + t,\ t^2 - 1,\ 1 - t)$ defined on $\langle -1, 1\rangle$, is maximal.
Solution: $t = -\sqrt[3]{\dfrac{1}{2}}, d = \sqrt{6 - \dfrac{3}{2}\sqrt[3]{\dfrac{1}{2}}}$

Problem 6: Determine angle of vectors that are values of two vector functions
$\boldsymbol{r}(t) = (a\cos t,\ b\sin t)$, $a, b \in R$, and $\boldsymbol{p}(t) = (c\sin t,\ d\cos t)$, $c, d \in R$, $t \in \langle 0, 2\pi\rangle$
in the point $t = \dfrac{\pi}{4}$ and calculate their vector product. Evaluate for $a = 3$, $b = 4$, $c = 5$, $d = 12$.

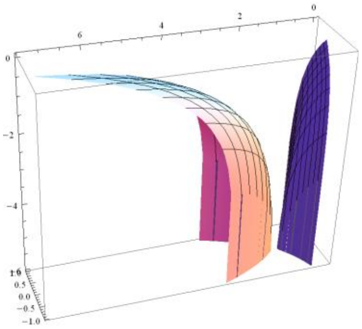
Solution:
$$\cos\varphi = \frac{ac + bc}{\sqrt{(a^2 + b^2)(c^2 + d^2)}}, \boldsymbol{r}(\frac{\pi}{4}) \times \boldsymbol{p}(\frac{\pi}{4}) = (0,0,\frac{ad - bc}{2})$$
$$\cos\varphi = \frac{65}{63}, \varphi = \arccos\frac{65}{63} = 0{,}25rd = 14^\circ 20', \boldsymbol{r}(\frac{\pi}{4}) \times \boldsymbol{p}(\frac{\pi}{4}) = (0,0,8)$$

Problem 7: Determine domain of definition of vector function
$\boldsymbol{r}(u, v) = (u, v,\ 2u/(u - v),\ (u + v)/(u - v))$. Find value of function in points
$(3,-3)$, $(3, 0)$, $(0,-3)$, and sketch part of function graph on region $\langle 0, 3\rangle \times \langle -3, 0\rangle$.
Solution: $D(\boldsymbol{r}) = \{(u, v) \in R^2,\ u \neq v\}$**,** $\boldsymbol{r}(3,-3) = (-9,1,0)$, $\boldsymbol{r}(3, 0) = (0,2,1)$, $\boldsymbol{r}(0,-3) = (0,0,-1)$.



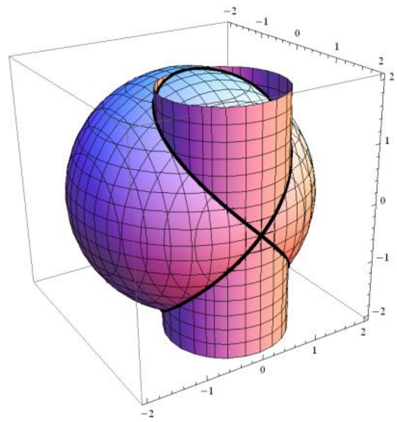
Problem 8: Determine domain of definition of vector function $\boldsymbol{r}(u, v) = (e^{u+v}, \ln(u.v), (u-v))$. Find values of function in points $(-1, -1)$, $(-1, 1)$, $(1,-1)$, $(1,1)$ and sketch part of function graph on region $\langle -1, 1\rangle^2$.

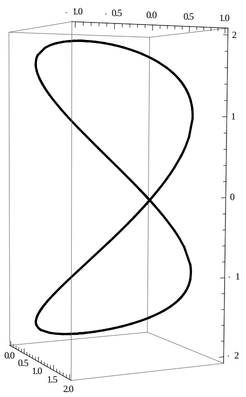
Solution: $D(\boldsymbol{r}) = \{(u, v) \in R^2 : (u > 0 \wedge v > 0) \vee (u < 0 \wedge v < 0)\}$
$\boldsymbol{r}(-1, -1) = (e^{-2}, 0, 0)$, $\boldsymbol{r}(1, 1) = (e^2, 0, 0)$, $\boldsymbol{r}(-1, 1)$ and $\boldsymbol{r}(1,-1)$ is not defined.



Problem 9: Viviani curve is intersection curve of sphere determined by implicite equation $x^2 + y^2 + z^2 = a^2$ and cylindrical surface of revolution determined by equation $x^2 + y^2 = ax$, while its view in the coordinate plane xy is a circle with the same equation $x^2 + y^2 = ax$. Find vector function, whose graph is Viviani curve, and calculate coordinates of its self-itersection point and intersection point with coordinate axis z. Visualize surface intersection together with the view of intersection curve.

Solution: $\boldsymbol{r}(t) = \left(\frac{a}{2}(1+\cos t), \frac{a}{2}\sin t, a\sin\frac{t}{2}\right), t \in \langle 0, 4\pi\rangle$, $P = [2, 0, 0]$





Problem 10: Find vector function, whose graph is a space cuve determined as intersection of hyperbolic paraboloid and cylindrical surface of revolution, that are determined by equations $z = xy$, $x^2 + y^2 = 1$. Calculate coordinates of curve intersections with coordinate axes x and y.

Solution: $\boldsymbol{r}(t) = (\cos t, \sin t, \cos t \sin t)$, $t \in \langle 0, 2\pi\rangle$, $X_{1,2} = [\pm 1,0,0]$, $Y_{1,2} = [0, \pm 1,0]$.