

PROBLEMS ON FOURIER SERIES

Problem 1: Find expansion of function f to Fourier series on given interval:

a) $f(x) = 2x, 0 < x < 1$

b) $f(x) = 110 - x, 105 < x < 115$

c) $f(x) = \frac{1}{\pi}(\pi - x), 0 < x < 2\pi$

d) $f(x) = \frac{\pi}{4} - \frac{x}{2}, -\pi < x < \pi$

e) $f(x) = x^3, 0 < x < \pi$

f) $f(x) = 1 - \frac{1}{\pi^2}(x - \pi)^2, 0 < x < 2\pi$

g) $f(x) = \cos \frac{x}{3}, -\pi < x < \pi$

h) $f(x) = \frac{1}{2}(x + |x|), -\pi < x < \pi$

i) $f(x) = (x - 2\pi)^2, \pi < x < 3\pi$

j) $f(x) = e^x, -\frac{\pi}{2} < x < \frac{\pi}{2}$

k) $f(x) = e^{-x}, -\pi < x < \pi$

l) $f(x) = -x, -\pi < x < \pi$

m) $f(x) = x, 0 < x < 1$

n) $f(x) = |x|, -\pi < x < \pi$

o) $f(x) = 1 - x, 0 < x < 1$

p) $f(x) = |\cos x|, 0 < x < 2\pi$

q) $f(x) = |\sin x|, 0 < x < 2$

Solutions: a) $\frac{1}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin 2n\pi x}{n}$

b) $\frac{10}{\pi} \sum_{n=1}^{\infty} (-1)^n \frac{\sin \frac{n\pi x}{5}}{n}$

c) $\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin nx}{n}$

d) $\frac{\pi}{4} + \sum_{n=1}^{\infty} (-1)^n \frac{\sin nx}{n}$

e) $\frac{\pi^3}{4} + \sum_{n=1}^{\infty} \frac{3\pi}{2n^2} \cos 2nx + \left(\frac{3}{2n^3} - \frac{\pi^2}{n} \right) \sin 2nx$

f) $\frac{2}{3} - \frac{4}{\pi^2} + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$

g) $\frac{3\sqrt{3}}{2\pi} + \frac{3\sqrt{3}}{2\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{1-9n^2} \cos nx$

h) $\frac{\pi}{4} - \sum_{n=1}^{\infty} \frac{2}{\pi(2n-1)^2} \cos(2n-1)x + \frac{(-1)^n}{n} \sin nx$

i) $\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$

j) $\frac{2}{\pi} \sinh \frac{\pi}{2} + \frac{4}{\pi} \sinh \frac{\pi}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{1+4n^2} (\cos 2nx + 2n \sin 2nx)$

k) $\frac{1}{\pi} \sinh \pi + 2 \sinh \pi \sum_{n=1}^{\infty} \frac{(-1)^n}{(1+n^2)\pi} (\cos nx + n \sin nx)$

l) $2 \sum_{n=1}^{\infty} (-1)^n \frac{\sin nx}{n}$

m) $\frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin 2n\pi x}{n}$

n) $\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2}$

o) $\frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin 2n\pi x}{n}$

p) $\frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1} \cos 2nx$

q) $\frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2-1} \cos 2nx$