

PROBLEMS ON VECTORS

Problem 1: Are vectors $\mathbf{a} = (-1, 2, 1)$ and $\mathbf{b} = (1, 0, 1)$ perpendicular? Why?

Solution: Yes, because $\mathbf{a} \cdot \mathbf{b} = 0$

Problem 2: Find vector perpendicular to vectors $\mathbf{a} = (1, 0, 1)$, $\mathbf{b} = (-1, 1, 0)$.

Solution: $\mathbf{a} \times \mathbf{b} = (-1, -1, 1)$

Problem 3: Are vectors $\mathbf{a} = (-1, 2, 4)$, $\mathbf{b} = (3, 0, 2)$, $\mathbf{c} = (-2, 1, 4)$ coplanar?

Solution: No, $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = -18$.

Problem 4: Find size of vectors $\mathbf{a} = (-1, 2, 2)$, $\mathbf{b} = (1, 0, 0)$, $\mathbf{c} = (3, -2, 6)$.

Solution: $|\mathbf{a}| = 3$, $|\mathbf{b}| = 1$, $|\mathbf{c}| = 7$

Problem 5: Find sum and difference of vectors $\mathbf{a} = (2, 1, 3)$, $\mathbf{b} = (1, -1, 0)$ and their size.

Solution: $\mathbf{a} + \mathbf{b} = (3, 0, 3)$, $|\mathbf{a} + \mathbf{b}| = 3\sqrt{2}$; $\mathbf{a} - \mathbf{b} = (1, 2, 3)$, $|\mathbf{a} - \mathbf{b}| = 2\sqrt{7}$,

Problem 6: Find angle formed by vectors $\mathbf{a} = (6, 0, 3)$ and $\mathbf{b} = (2, 5, 4)$.

Solution: $\alpha = 138^\circ$

Problem 7: Find angle formed by vectors $\mathbf{a}$, $\mathbf{b}$, while: vector $\mathbf{a} + 3\mathbf{b}$ is perpendicular to vector $7\mathbf{a} - 5\mathbf{b}$ and vector $\mathbf{a} - 4\mathbf{b}$ is perpendicular to vector $7\mathbf{a} - 2\mathbf{b}$.

Solution: $\alpha = 60^\circ$

Problem 8: Find area of parallelogram $ABCD$ with given vertices

$A = [0, 4, 7]$, $B = [2, 2, 2]$, $D = [6, 1, 5]$.

Solution: $P = \sqrt{833}$

Problem 9: Calculate area of triangle $A = [3, 1, 4]$, $B = [0, 2, 1]$, $C = [5, 0, 8]$.

Solution: $P = \sqrt{38} / 2$

Problem 10: Find, whether 4-gon $ABCD$ is a square, when

$A = [5, 2, 6]$, $B = [6, 4, 4]$, $C = [4, 3, 2]$, $D = [3, 1, 4]$.

Solution: Yes.

$\mathbf{a} = AB$, $\mathbf{b} = BC$, $\mathbf{c} = CD$, $\mathbf{d} = DA$, $\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow \mathbf{a} \perp \mathbf{b}$, $\mathbf{a} \cdot \mathbf{d} = 0 \Rightarrow \mathbf{a} \perp \mathbf{d}$, $\mathbf{a} \cdot \mathbf{c} = 0 \Rightarrow \mathbf{a} \perp \mathbf{c}$,
 $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = |\mathbf{d}| = 3$.

Problem 11: Evaluate volume of parallelepiped $ABCD A'B'C'D'$ with vertices

$A = [0, 0, 0]$, $B = [0, 4, 4]$, $C = [3, 4, 0]$, $E = [5, 0, 3]$.

Solution: 116

Problem 12: Evaluate volume of tetrahedron with vertices

$A = [1, -5, 4]$, $B = [0, 3, 1]$, $C = [-2, -4, 3]$, $D = [-4, 4, -2]$.

Solution: 41/6