ÚLOHY O VEKTOROVÝCH FUNKCIÁCH III

Príklad 1: Dané sú vektorové funkcie

a) $\boldsymbol{r}(t) = (\sin t, \cos t, 3 + t^2)$, $\boldsymbol{p}(t) = (2t, -t^3, t^2 - 1)$

b) $\boldsymbol{r}(t) = (\cos t, 1/\cos t, \sin t)$, $\boldsymbol{p}(t) = (\sin t, \cos t, \tan t)$

c) $\boldsymbol{r}(t) = (e^t, e^{2t}, t.e^t)$, $\boldsymbol{p}(t) = (e^{-t}, te^t, e^t)$

d) $\boldsymbol{r}(t) = (\ln(t^2 + 2t + 1), \ln(t + 2), t.\ln^{-1}(t + 1))$, $\boldsymbol{p}(t) = (\ln(t + 1)^{-1}, \ln^{-1}(t + 2), t.\ln(t + 1))$

Zistite, čomu sa rovná ich skalárny a vektorový súčin v bode $t = 0$.

Riešenie: a) $\boldsymbol{r}(0) = (0,1,3)$, $\boldsymbol{p}(0) = (0,0,-1)$, $\boldsymbol{r}(t).\boldsymbol{p}(t) = 2t\sin t - t^3\cos t + t^4 + t^2 - 3$, $\boldsymbol{r}(0).\boldsymbol{p}(0) = -3$,

$\boldsymbol{r}(t) \times \boldsymbol{p}(t) = ((t^2 - 1)\cos t - t^3(t^2 + 3), -(t^2 - 1)\sin t + 2t(3 + t^2), -t^3\sin t - 2t\cos t)$,

$\boldsymbol{r}(0) \times \boldsymbol{p}(0) = (-1, 0, 0)$

b) $\boldsymbol{r}(0) = (1, 1, 0)$,.$\boldsymbol{p}(0) = (0, 1, 0)$, $\boldsymbol{r}(t).\boldsymbol{p}(t) = 1 + \cos t\sin t + \sin t\tan t$, $\boldsymbol{r}(0).\boldsymbol{p}(0) = 1$,

$\boldsymbol{r}(t) \times \boldsymbol{p}(t) = (-\sin t\cos t + \sec t\tan t, -\sin t + \sin^2 t, \cos^2 t - \tan t)$,

$\boldsymbol{r}(0) \times \boldsymbol{p}(0) = (0, 0, 1)$

c) $\boldsymbol{r}(0) = (1, 1, 0)$,.$\boldsymbol{p}(0) = (1, 0, 1)$, $\boldsymbol{r}(t).\boldsymbol{p}(t) = 1 + te^{3t} + te^{2t}$, $\boldsymbol{r}(0).\boldsymbol{p}(0) = 1$,

$\boldsymbol{r}(t) \times \boldsymbol{p}(t) = (e^{3t} - t^2e^{2t}, e^{2t} - t, te^{2t} - e^t)$, $\boldsymbol{r}(0) \times \boldsymbol{p}(0) = (1, -1, -1)$

d) $\boldsymbol{r}(0) = (0, \ln 2, 0)$,.$\boldsymbol{p}(0) = (0, 1/\ln 2, 0)$, $\boldsymbol{r}(t).\boldsymbol{p}(t) = 1 + t^2 + \ln(t^2 + 2t + 1) . \ln(t + 1)^{-1}$,

$\boldsymbol{r}(0).\boldsymbol{p}(0) = 1$,

$\boldsymbol{r}(t) \times \boldsymbol{p}(t) = (t.\ln(t + 1).\ln(t + 2) - t.\ln^{-1}(t + 1). \ln^{-1}(t + 2),$

$t.\ln(t + 1). \ln(t^2 + 2t + 1) - t.\ln(t + 1)^{-1}.\ln^{-1}(t + 1),$

$\ln(t^2 + 2t + 1).\ln^{-1}(t + 2) - \ln(t + 2). \ln(t + 1)^{-1}),$

$\boldsymbol{r}(0) \times \boldsymbol{p}(0) = (0, 0, 0)$

Príklad 2: Nájdite súčet $\boldsymbol{s}(t)$ a rozdiel $\boldsymbol{d}(t)$ vektorových funkcií $\boldsymbol{r}(t) = (2t, t^3 + t - 1, 1 + t^2)$, $\boldsymbol{p}(t) = (2t, 1 - t^3, t - t^2)$ a zistite, čomu sa rovná ich skalárny a vektorový súčin funkcií $\boldsymbol{s}(t)$ a $\boldsymbol{d}(t)$ v bode $t = 1$. Overte platnosť rovností

$$\boldsymbol{s}(t).\boldsymbol{d}(t) = (\boldsymbol{r}(t) + \boldsymbol{p}(t)).(\boldsymbol{r}(t) - \boldsymbol{p}(t)) = \boldsymbol{r}(t).\boldsymbol{r}(t) - \boldsymbol{p}(t).\boldsymbol{p}(t)$$

$$\boldsymbol{s}(t) \times \boldsymbol{d}(t) = (\boldsymbol{r}(t) + \boldsymbol{p}(t)) \times (\boldsymbol{r}(t) - \boldsymbol{p}(t)) = \boldsymbol{p}(t) \times \boldsymbol{r}(t) - \boldsymbol{r}(t) \times \boldsymbol{p}(t)$$

Riešenie: $\boldsymbol{s}(t) = \boldsymbol{r}(t) + \boldsymbol{p}(t) = (4t, t, 1 + t)$, $\boldsymbol{d}(t) = \boldsymbol{r}(t) - \boldsymbol{p}(t) = (0, 2t^3 + t - 2, 2t^2 - t + 1)$,

$\boldsymbol{s}(t).\boldsymbol{d}(t) = 2t^4 + 2t^3 + 2t^2 - 2t + 1$, $\boldsymbol{s}(1) = (4, 1, 2)$,.$\boldsymbol{d}(1) = (0, 1, 2)$, $\boldsymbol{s}(1).\boldsymbol{d}(1) = 5$

$\boldsymbol{r}(t).\boldsymbol{r}(t) - \boldsymbol{p}(t).\boldsymbol{p}(t) = t^6 + 3t^4 - 2t^3 + 7t^2 - 2t + 2 - (t^6 + t^4 - 4t^3 + 5t^2 + 1) = 2t^4 + 2t^3 + 2t^2 - 2t + 1$

$\boldsymbol{s}(t) \times \boldsymbol{d}(t) = (-2t^4 - 2t^2 + 2t + 2, -8t^3 + 4t^2 - 4t, 8t^3 + 4t^2 - 8t)$, $\boldsymbol{s}(1) \times \boldsymbol{d}(1) = (0, 8, -4)$

$\boldsymbol{p}(t) \times \boldsymbol{r}(t) - \boldsymbol{r}(t) \times \boldsymbol{p}(t) =$

$= (-t^4 - t^2 + t + 1, -4t^3 + 2t^2 - 2t, 4t^4 + 2t^2 - 4t) - (t^4 + t^2 - t - 1, 4t^3 - 2t^2 + 2t, -4t^4 - 2t^2 + 4t) =$

$= (-2t^4 - 2t^2 + 2t + 2, -8t^3 + 4t^2 - 4t, 8t^3 + 4t^2 - 8t)$, $\boldsymbol{p}(t) \times \boldsymbol{r}(t) - \boldsymbol{r}(t) \times \boldsymbol{p}(t) = (0, -8, 4)$

Príklad 3: Nájdite prvé a druhé derivácie skalárneho a vektorového súčinu vektorových funkcií

a) $\boldsymbol{r}(t) = (2t, t^2 - 1, 1 - t)$, $\boldsymbol{p}(t) = (-2t, t - 1, t)$

b) $\boldsymbol{r}(t) = (\sin 2t, 2\sin t, \cos t)$, $\mathbf{p}(t) = (2\cos t, \cos 2t, \sin 2t)$

c) $\boldsymbol{r}(t) = (\ln(t + 1), \ln(t + 1)^{-1}, 1)$, $\mathbf{p}(t) = (\ln(t + 1)^{-1}, 1, \ln(t + 1))$

d) $\boldsymbol{r}(t) = (\sin t, \cos t, 1)$, $\mathbf{p}(t) = (\operatorname{tg} t, \operatorname{cotg} t, a^t)$

a zistite ich hodnoty v bode $t = 0$.

Riešenie:.a) $[\boldsymbol{r}(t).\boldsymbol{p}(t)]' = 3t^2 - 6t + 1$, $[\boldsymbol{r}(1).\boldsymbol{p}(1)]' = 1$

$[\boldsymbol{r}(t).\boldsymbol{p}(t)]'' = 6t - 6$, $[\boldsymbol{r}(1).\boldsymbol{p}(1)]'' = -6$

$[\mathbf{r}(t) \times \mathbf{p}(t)]' = (3t^2 + 2t - 3, -2, 6t^2 + 4t - 4)$, $[\mathbf{r}(t) \times \mathbf{p}(t)]'_{t=0} = (-3, -2, -4)$

$[\mathbf{r}(t) \times \mathbf{p}(t)]'' = (6t + 2, 0, 12t + 4)$, $[\mathbf{r}(t) \times \mathbf{p}(t)]''_{t=0} = (2, 0, 4)$

b) $[\boldsymbol{r}(t).\boldsymbol{p}(t)]' = -7\sin t \sin 2t + 8\cos t \cos 2t$, $[\boldsymbol{r}(0).\boldsymbol{p}(0)]' = 8$

$[\boldsymbol{r}(t).\boldsymbol{p}(t)]'' = -22\sin t \cos 2t - 23\cos t \sin 2t$, $[\boldsymbol{r}(0).\boldsymbol{p}(0)]'' = 0$

$[\mathbf{r}(t)\times\mathbf{p}(t)]' = (4\cos t \sin 2t + 5\sin t \cos 2t,\ 4(-\sin 2t \cos 2t + \cos t \sin t),\ 2(\cos^2 2t - \sin^2 2t) - 4(\cos^2 t - \sin^2 t))$,

$[\mathbf{r}(t) \times \mathbf{p}(t)]'_{t=0} = (0, 0, -2)$

$[\mathbf{r}(t)\times\mathbf{p}(t)]'' = (-14\sin t \sin 2t + 13\cos t \cos 2t, 8(\sin^2 2t - \cos^2 2t) + 4(\cos^2 t - \sin^2 t), 16(\sin t \cos t - \sin 2t \cos 2t)$,

$[\mathbf{r}(t) \times \mathbf{p}(t)]''_{t=0} = (13, -4, 0)$

c) $[\boldsymbol{r}(t).\boldsymbol{p}(t)]' = (t+1)^{-1} \ln(t+1)^{-1} + (t+1)\ln(t+1) + t + 1 + (t+1)^{-1}$,

$[\boldsymbol{r}(t).\boldsymbol{p}(t)]'_{t=0} = 2$

$[\boldsymbol{r}(t).\boldsymbol{p}(t)]'' = -(t+1)^{-2}\ln(t+1)^{-1} + \ln(t+1) + 3 - (t+1)^{-1}$, $[\boldsymbol{r}(t).\boldsymbol{p}(t)]'_{t=0} = 2$

$[\mathbf{r}(t) \times \mathbf{p}(t)]' = ((t+1)\ln(t+1) + (t+1)^{-1}\ln(t+1)^{-1}, 2(t+1)^{-1}\ln(t+1) - (t+1), (t+1)^{-1}\ln^2(t+1)^{-1}$
$+2(t+1)\ln(t+1)\ln(t+1)^{-1})$, $[\mathbf{r}(t) \times \mathbf{p}(t)]'_{t=0} = (0, -1, 0))$

$[\mathbf{r}(t) \times \mathbf{p}(t)]''_{t=0} = (1, 2, 2)$

d) $[\boldsymbol{r}(t).\boldsymbol{p}(t)]' = 2(\sin t - \cos t) + \sin^3 t \cos^{-2} t + \cos^3 t \sin^{-2} t + a^t \ln a$,

$[\boldsymbol{r}(t).\boldsymbol{p}(t)]'_{t=0} = -2 + \ln a$

$[\boldsymbol{r}(t).\boldsymbol{p}(t)]'' = 2(\sin t + \cos t) + \sin^2 t \cos^{-3} t(3\cos^2 t + \sin^2 t) + \sin^{-3} t \cos^2 t(3\sin^2 t + \cos^2 t) + a^t \ln^2 a$,

$[\boldsymbol{r}(t).\boldsymbol{p}(t)]'_{t=0} = 2 + \ln^2 a$

$[\mathbf{r}(t) \times \mathbf{p}(t)]' = (a^t(\ln a \cos t - \sin t) + \sin^{-2} t,\ a^t(\ln a \sin t + \cos t) - \cos^{-2} t,\ \cos t - \sin t)$,

$[\mathbf{r}(t) \times \mathbf{p}(t)]'_{t=0} = (\ln a, 0, 1)$

$[\mathbf{r}(t) \times \mathbf{p}(t)]''_{t=0} = (\ln^2 a - 1, 2 \ln a, -1)$

Príklad 4: Nájdite parciálne derivácie vektorových funkcií dvoch premenných do rádu 2

a) $\boldsymbol{f}(u,v) = (u.v,\ u + v,\ uv^{-1})$, b) $\boldsymbol{f}(u,v) = (\sin u \sin v, \sin u \cos v, u.\cos v)$

c) $\boldsymbol{f}(u,v) = (e^{u.v}, e^{u+v}, e^{u/v})$, d) $\boldsymbol{f}(u,v) = (\ln u \ln v, \arcsin u \arccos v, \tan u. \arctan v)$

Riešenie: a) $\boldsymbol{f}_u(u,v) = (v, 1, v^{-1})$, $\boldsymbol{f}_v(u,v) = (u, 1, -uv^{-2})$,

$\boldsymbol{f}_{uu}(u,v) = (0, 0, 0)$, $\boldsymbol{f}_{vv}(u,v) = (0, 0, 2uv^{-3})$, $\boldsymbol{f}_{uv}(u,v) = (1, 0, -v^{-2})$, $\boldsymbol{f}_{vu}(u,v) = (u, 1, -v^{-2})$

b) $\boldsymbol{f}_u(u,v) = (\cos u \sin v, \cos u \cos v, \cos v)$, $\boldsymbol{f}_v(u,v) = (\sin u \cos v, -\sin u \sin v, -u \sin v)$

$\boldsymbol{f}_{uu}(u,v) = (-\sin u \sin v, -\sin u \cos v, 0)$, $\boldsymbol{f}_{vv}(u,v) = (-\sin u \sin v, -\sin u \cos v, -u \cos v)$

$\boldsymbol{f}_{uv}(u,v) = (\cos u \cos v, -\cos u \sin v, -\sin v)$, $\boldsymbol{f}_{vu}(u,v) = (\cos u \cos v, -\mathrm{con}\ u \sin v, -\sin v)$

c) $\boldsymbol{f}_u(u,v) = (ve^{u.v}, e^{u+v}, v^{-1}e^{u/v})$, $\boldsymbol{f}_v(u,v) = (ue^{u.v}, e^{u+v}, -uv^{-2}e^{u/v})$

$\boldsymbol{f}_{uu}(u,v) = (v^2e^{u.v}, e^{u+v}, v^{-2}e^{u/v})$, $\boldsymbol{f}_{vv}(u,v) = (u^2e^{u.v}, e^{u+v}, uv^{-3}e^{u/v}(v^{-1}+2))$

$\boldsymbol{f}_{uv}(u,v) = (e^{u.v}(1+uv), e^{u+v}, -v^{-2}e^{u/v}(1+uv^{-1}))$, $\boldsymbol{f}_{vu}(u,v) = (e^{u.v}(1+uv), e^{u+v}, -v^{-2}e^{u/v}(1+uv^{-1})$

d) $$\boldsymbol{f}_u(u,v) = \left(\frac{\ln v}{u}, \frac{\arccos v}{\sqrt{1-u^2}}, \frac{\arctan v}{\cos^2 u}\right), \boldsymbol{f}_v(u,v) = \left(\frac{\ln u}{v}, \frac{-\arcsin u}{\sqrt{1-v^2}}, \frac{\tan u}{1+v^2}\right)$$

$$\boldsymbol{f}_{uu}(u,v) = \left(-\frac{\ln v}{u^2}, \frac{u \arccos v}{\sqrt{(1-u^2)^3}}, \frac{2\sin u \arctan v}{\cos^3 u}\right), \boldsymbol{f}_{vv}(u,v) = \left(-\frac{\ln u}{v^2}, \frac{v\mathrm{ar}c\sin u}{\sqrt{(1-v^2)^3}}, \frac{-2v\tan u}{(1+v^2)^2}\right)$$

$$\boldsymbol{f}_{uv}(u,v) = \boldsymbol{f}_{vu}(u,v) = \left(\frac{1}{uv}, \frac{-1}{\sqrt{(1-u^2)(1-v^2)}}, \frac{1}{(1+v^2)\cos^2 u}\right)$$

Príklad 5: Nájdite prvé parciálne derivácie vektorových funkcií dvoch premenných a v bode $(u, v) = (1, 1)$ vypočítajte ich hodnoty a hodnoty ich skalárneho a vektorového súčinu:

a) $\boldsymbol{f}(u, v) = (-2u, v - 1, u + v)$

b) $\boldsymbol{f}(u, v) = (u - v^2, 1 - u^3, 3v - 2)$

c) $\boldsymbol{f}(u, v) = (\ln(u + v), \ln(u^2 + v^2), \ln uv)$

d) $\boldsymbol{f}(u, v) = (e^u + e^v, e^u e^v, e^u e^{-v})$

Riešenie:.a) $\boldsymbol{f}_u(u, v) = (-2, 0, 1)$, $\boldsymbol{f}_v(u, v) = (0, 1, 1)$,
$\boldsymbol{f}_u(u, v).\ \boldsymbol{f}_v(u, v) = 1$, $\boldsymbol{f}_u(u, v) \times \boldsymbol{f}_v(u, v) = (-1, 2, -2)$

b) $\boldsymbol{f}_u(u, v) = (1,-3u^2, 0)$, $\boldsymbol{f}_v(u, v) = (-2v, 0, 3)$, $\boldsymbol{f}_u(1, 1) = (1,-3, 0)$, $\boldsymbol{f}_v(1, 1) = (-2, 0, 3)$,
$\boldsymbol{f}_u(u, v).\ \boldsymbol{f}_v(u, v) = -2v$, $\boldsymbol{f}_u(1, 1).\ \boldsymbol{f}_v(1, 1) = -2$,
$\boldsymbol{f}_u(u, v) \times \boldsymbol{f}_v(u, v) = (-9u^2,-3, -6u^2v)$, $\boldsymbol{f}_u(1, 1) \times \boldsymbol{f}_v(1, 1) = (-9,-3, -6)$

c) $\boldsymbol{f}_u(u, v) = ((u + v)^{-1}, 2u(u^2 + v^2)^{-1}, u^{-1})$, $\boldsymbol{f}_v(u, v) = ((u + v)^{-1}, 2v(u^2 + v^2)^{-1}, v^{-1})$,
$\boldsymbol{f}_u(1, 1) = (1/2, 1, 1)$, $\boldsymbol{f}_v(1, 1) = (1/2, 1, 1)$,
$\boldsymbol{f}_u(u, v).\ \boldsymbol{f}_v(u, v) = (u^2 + v^2)^{-2} + 4uv(u^2 + v^2)^2 + (uv)^{-1}$, $\boldsymbol{f}_u(1, 1).\ \boldsymbol{f}_v(1, 1) = 9/4$,
$\boldsymbol{f}_u(u, v) \times \boldsymbol{f}_v(u, v) = (2(u^2-v^2)(uv(u^2 + v^2))^{-1}, (u-v)(uv(u+v))^{-1}, 2(v-u)((u+v)(u^2 + v^2))^{-1})$,
$\boldsymbol{f}_u(1, 1) \times \boldsymbol{f}_v(1, 1) = (0, 0, 0)$

d) $\boldsymbol{f}_u(u, v) = (e^u, e^{u+v}, e^{u-v})$, $\boldsymbol{f}_v(u, v) = (e^v, e^{u+v}, -e^{u-v})$,
$\boldsymbol{f}_u(1, 1) = (e, e^2, 1)$, $\boldsymbol{f}_v(1, 1) = (e, e^2, -1)$,
$\boldsymbol{f}_u(u, v).\ \boldsymbol{f}_v(u, v) = e^{u+v} + e^{2(u+v)} - e^{2(u-v)}$, $\boldsymbol{f}_u(1, 1).\ \boldsymbol{f}_v(1, 1) = e^2 + e^4 - 1$
$\boldsymbol{f}_u(u, v) \times \boldsymbol{f}_v(u, v) = (-2e^{2u}, e^{2u-v} + e^u, e^{2u+v} - e^{u+2v})$,
$\boldsymbol{f}_u(1, 1) \times \boldsymbol{f}_v(1, 1) = (-2e^2, 2e, 0)$

Príklad 6: Nájdite druhé parciálne derivácie vektorových funkcií dvoch premenných a ich zmiešaný súčin a vypočítajte jeho hodnotu bode (0, 0).

a) $\boldsymbol{f}(u, v) = (u.v, u + v, (u+1)^{v+1})$
b) $\boldsymbol{f}(u, v) = (\sin u \sin v, \sin u \cos v, u.\cos v)$
c) $\boldsymbol{f}(u, v) = (e^{u.v}, e^{u+v}, e^{u/v})$
d) $\boldsymbol{f}(u, v) = (\ln(u+1) \ln(v+1), \arcsin u \arccos v, \tan u.\ \arctan v)$

Riešenie: a) $\boldsymbol{f}_{uu}(u, v) = (0, 0, (u+1)^{v-1}v(v+1)$, $\boldsymbol{f}_{uv}(u, v) = (1, 0, (u+1)^{\mathrm{v}}(1+(1+\mathrm{v})\log(1+u)))$,
$\boldsymbol{f}_{vv}(u, v) = (0, 0, (u+1)^{v+1}\ln^2 (u+1))$
$[\boldsymbol{f}_{uu}(u, v), \boldsymbol{f}_{uv}(u, v), \boldsymbol{f}_{vv}(u, v)] = 0$

b) $\boldsymbol{f}_{uu}(u, v) = (-\sin u \sin v, -\sin u \cos v, 0)$, $\boldsymbol{f}_{uv}(u, v) = (\cos u \cos v, -\cos u \sin v, -\sin v)$
$\boldsymbol{f}_{vv}(u, v) = (-\sin u \sin v, -\sin u \cos v, - u\cos v)$
$[\boldsymbol{f}_{uu}(u, v), \boldsymbol{f}_{uv}(u, v), \boldsymbol{f}_{vv}(u, v)] = - u \sin u \cos u \ \cos v$
$[\boldsymbol{f}_{uu}(0, 0), \boldsymbol{f}_{uv}(0, 0), \boldsymbol{f}_{vv}(0, 0)] = 0$

c) $\boldsymbol{f}_{uu}(u,v) = (v^2e^{u.v}, e^{u+v}, e^{u/v}/v^2)$, $\boldsymbol{f}_{uv}(u,v) = ((1+uv)e^{u.v}, e^{u+v}, -(u+v)e^{u/v}/v^3)$,
$\boldsymbol{f}_{vv}(u,v) = (u^2e^{u.v}, e^{u+v}, u(u+2v)e^{u/v}/v^4)$
$\boldsymbol{f}_{uu}(u, v), \boldsymbol{f}_{uv}(u, v), \boldsymbol{f}_{vv}(u, v)] = (-2u^3v + v^2 + v^4 - u^2(1+3v^2)+2uv(2\mathrm{v}^2-1))e^{v+u(1+v+1/v)}/v^4$
$[\boldsymbol{f}_{uu}(0, 0), \boldsymbol{f}_{uv}(0, 0), \boldsymbol{f}_{vv}(0, 0)] = 0$

d) $$\boldsymbol{f}_{uu}(u,v) = \left(-\frac{\ln(1+v)}{(1+u)^2}, \frac{u\arccos v}{\sqrt{(1-u^2)^3}}, 2\arctan v \sec^2 u \tan u \right),$$

$$\boldsymbol{f}_{uv}(u,v) = \left(\frac{1}{(1+u)(1+v)}, \frac{-1}{\sqrt{1-u^2}\sqrt{1-v^2}}, \frac{\sec^2 u}{1+v^2} \right),$$

$$\boldsymbol{f}_{vv}(u,v) = \left(\frac{-\ln(1+u)}{(1+v)^2}, \frac{-v*\arcsin u}{\sqrt{(1-v^2)^3}}, \frac{2v\tan u}{(1+v^2)^2} \right),$$

$[\boldsymbol{f}_{uu}(0, 0), \boldsymbol{f}_{uv}(0, 0), \boldsymbol{f}_{vv}(0, 0)] = 0$

Príklad 7. Nájdite prvé parciálne derivácie vektorových funkcií troch premenných podľa všetkých premenných a v bode $(u, v, w) = (1, 1, 0)$ vypočítajte hodnotu ich zmiešaného súčinu:

a) $\mathbf{p}(u, v, w) = (2u^3, v^3 - 1, \sin w)$

b) $\mathbf{p}(u, v, w) = (u - v^2, v - w^3, uvw)$

c) $\mathbf{p}(u, v, w) = (\ln(u + w), e^{u+w} - v, e^v(u - w))$

d) $\mathbf{p}(u, v, w) = (\sin u + \sin v + \sin w, w\cos u \cos v, \sin (u v^{-1}) - \cos(vw^{-1}))$, $(u, v, w) = (\pi,\pi,\pi)$.

Riešenie:.a) $\boldsymbol{p}_u(u, v, w) = (6u^2,0,0)$, $\boldsymbol{p}_v(u, v, w) = (0, 3v^2,0)$, $\boldsymbol{p}_w(u, v, w) = (0,0, \cos w)$,

$[\boldsymbol{p}_u(u, v, w), \boldsymbol{p}_v(u, v, w), \boldsymbol{p}_w(u, v, w)] = 18u^2v^2 \cos w$,

$[\boldsymbol{p}_u(1,1,0), \boldsymbol{p}_v(1,1,0), \boldsymbol{p}_w(1,1,0)] = [(3,0,0), (0,3,0), (0,0,1)] = 18$

b) $\mathbf{p}_u(u, v, w) = (1, 0, vw)$, $\mathbf{p}_v(u, v, w) = (-2v, 1, uw)$, $\mathbf{p}_w(u, v, w) = (0,-3w^2, uv)$

$[\boldsymbol{p}_u(u, v, w), \boldsymbol{p}_v(u, v, w), \boldsymbol{p}_w(u, v, w)] = uv + 3w^3(u + 2v^2)$

$[\boldsymbol{p}_u(1,1,0), \boldsymbol{p}_v(1,1,0), \boldsymbol{p}_w(1,1,0)] = [(1,0,0), (-1,1,0), (0,0,1)] = 1$

c) $\mathbf{p}_u(u, v, w) = (1/(u + w), e^{u+w}, e^v)$, $\mathbf{p}_v(u, v, w) = (0,-1, e^v(u - w))$,

$\mathbf{p}_w(u, v, w) = (1/(u +w), e^{u+w}, -e^v)$,

$[\boldsymbol{p}_u(u, v, w), \boldsymbol{p}_v(u, v, w), \boldsymbol{p}_w(u, v, w)] = 2e^v /(u + w)$

$[\boldsymbol{p}_u(1,1,0), \boldsymbol{p}_v(1,1,0), \boldsymbol{p}_w(1,1,0)] = [(1,e, e), (0, -1, e), (1, e, -e)] = 2e$

d) $\mathbf{p}_u(u, v, w) = (\cos u, -w\sin u \cos v, v^{-1}\cos (uv^{-1}))$,

$\mathbf{p}_v(u, v, w) = (\cos v, -w\cos u \sin v, - uv^{-2}\cos (uv^{-1}) + w^{-1}\sin(vw^{-1}))$

$\mathbf{p}_w(u, v, w) = (\cos w, \cos u \cos v, - vw^{-2}\sin(vw^{-1})/)$

$[\boldsymbol{p}_u(\pi,\pi,\pi), \boldsymbol{p}_v(\pi,\pi,\pi), \boldsymbol{p}_w(\pi,\pi,\pi)] =$

$= [(-1, 0, \pi^{-1}\cos 1), (-1, 0, \pi^{-1}(\cos 1 + \sin 1), (-1, 1, -\pi^{-1}\sin 1)] = \pi^{-1}\sin 1$