- **Vypočítajte:**
  vektorový súčet, vektorový súčin a skalárny súčin vektorov
  a = (2,2,3), b = (2,-1,0),
  zmiešaný súčin vektorov a, b a c = (1, -1, 1)
  a veľkosť vektora a !

```
a = {2, 2, 3}; b = {2, -1, 0}; c = {1, -1, 1};
```

```
a + b
```

```
{4, 1, 3}
```

```
a.b
```

```
2
```

```
Cross[a, b]
```

```
{3, 6, -6}
```

```
MatrixForm[{a, b, c}]
```

$$\begin{pmatrix} 2 & 2 & 3 \\ 2 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$

```
Det[{a, b, c}]
```

```
-9
```

```
Norm[a]
```

$$\sqrt{17}$$

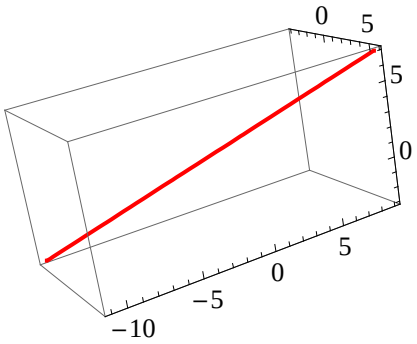
- **Napíšte parametrické rovnice priamky, ktorá prechádza bodmi A=[1,-1,2], B=[2,1,3]. Priamku znázornite v PSS.**

```
A = {1, -1, 2}; B = {2, 1, 3};
```

```
p = A + (B - A) * t
```

```
{1 + t, -1 + 2 t, 2 + t}
```

```
krp = ParametricPlot3D[p, {t, -5, 5},
  PlotStyle → {RGBColor[1, 0, 0], Thickness[0.01]}]
```

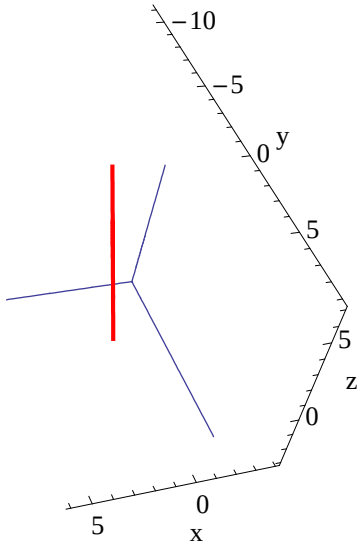



```
osx = ParametricPlot3D[{t, 0, 0}, {t, 0, 10}];
```

```
osy = ParametricPlot3D[{0, t, 0}, {t, 0, 10}];
```

```
osz = ParametricPlot3D[{0, 0, t}, {t, 0, 10}];
```

```
Show[krp, osx, osy, osz, Boxed -> False,
 AxesLabel → {"x", "y", "z"}]
```



- **Napíšte všeobecnú rovnicu roviny, ktorá prechádza bodom A=[1,3,0] a má normálový vektor n=(1,2,4). Rovinu znázornite v PSS.**

```
A = {1, 3, 0}; n = {1, 2, 4};
```

```
X = {x, y, z};
```

```
ρ = (X - A).n
```

$$-1 + x + 2\,(-3 + y) + 4\,z$$

```
ρ = (X - A).n // Simplify
```

$$-7 + x + 2\,y + 4\,z$$

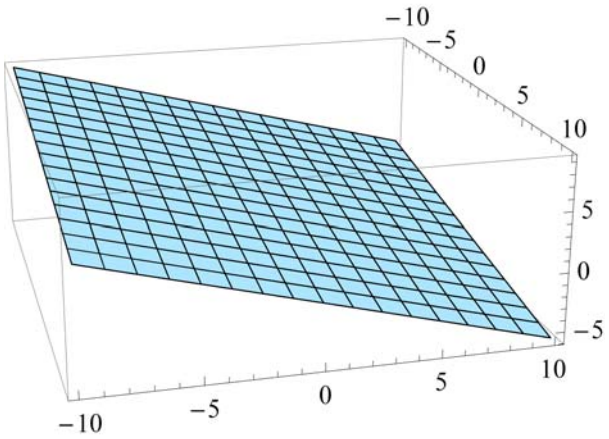
```
rov = Solve[ρ == 0, z]
```

$$\left\{\left\{z \to \frac{1}{4}\,(7 - x - 2\,y)\right\}\right\}$$

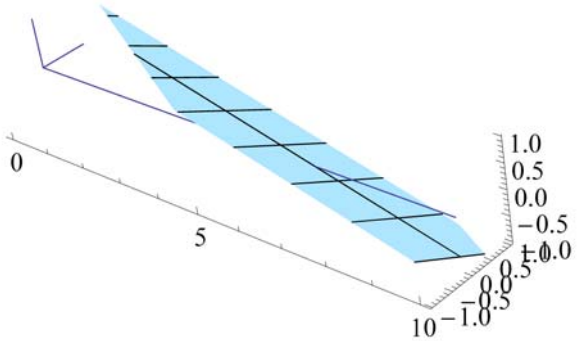
```
zsur = z /. rov[[1]]
```

$$\frac{1}{4}\,(7 - x - 2\,y)$$

```
gr = Plot3D[zsur, {x, -10, 10}, {y, -10, 10}]
```




```
Show[osx, osy, osz, gr, Boxed → False]
```



- **Napíšte rovnicu roviny, ktorá prechádza bodmi**
  P = [5,-1,1],
  Q = [-4,8,1],
  R = [0,0,5].

```
P = {5, -1, 1}; Q = {-4, 8, 1}; R = {0, 0, 5}; X = {x, y, z};
```

```
X - P
```

```
{-5 + x, 1 + y, -1 + z}
```

```
Q - P
```

```
{-9, 9, 0}
```

```
R - P
```

```
{-5, 1, 4}
```

```
r = Det[{X - P, Q - P, R - P}] == 0
```

```
-180 + 36 x + 36 y + 36 z == 0
```

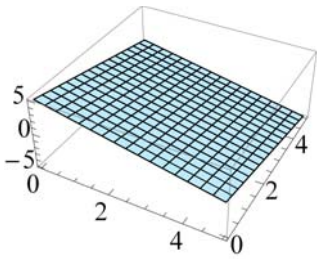
```
Solve[r, z]
```

```
{{z → 5 - x - y}}
```

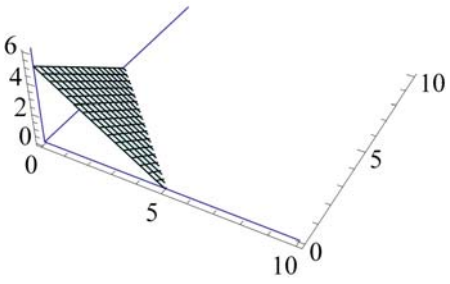
```
zsur = z /. %[[1]]
```

```
5 - x - y
```

```
gr = Plot3D[zsur, {x, 0, 5}, {y, 0, 5}]
```

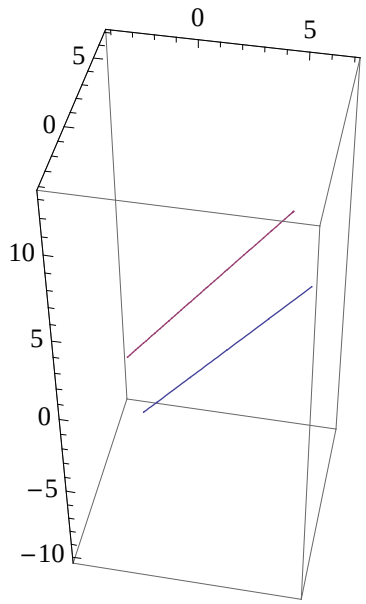



```
Show[gr, osx, osy, osz, Boxed -> False, PlotRange → {0, 6}]
```

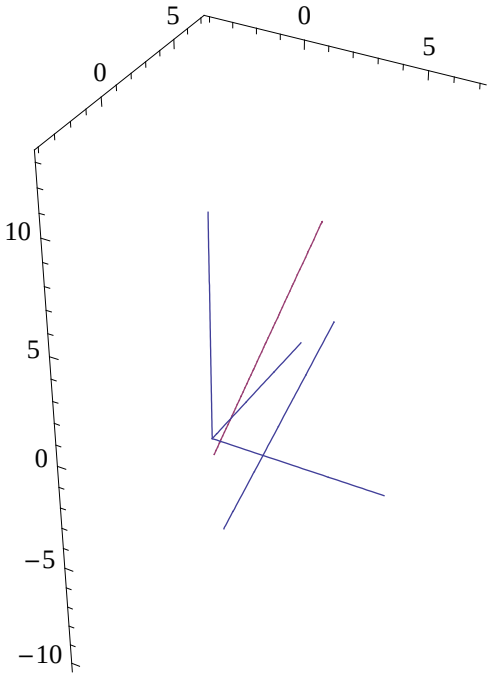


- **Zistite vzájomnú polohu priamok**
  p: x = 2 + t, y = 1 - t, z = 2t ,
  q: x = 1 - t, y = 2 + t, z = 3 - 2t .
  Ak ležia v jednej rovine, tak napíšte rovnicu roviny,
  ktorá je nimi určená.

```
gpq = ParametricPlot3D[{{2 + t, 1 - t, 2 t}, {1 - t, 2 + t, 3 - 2 t}},
  {t, -5, 5}]
```




```
Show[gpq, osx, osy, osz, Boxed → False]
```

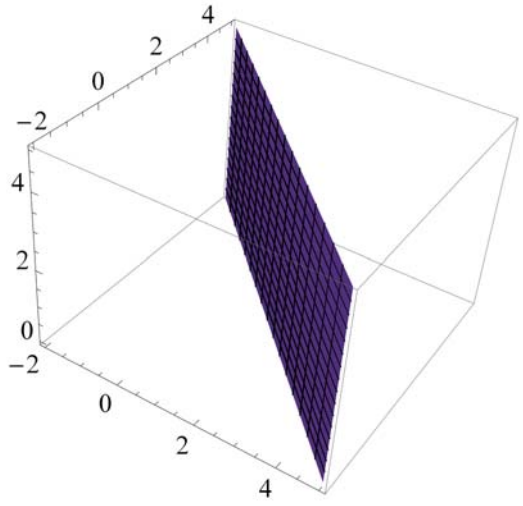



```
A = {2, 1, 0}; B = {1, 2, 3};

s = {1, -1, 2};

X = {x, y, z};
```

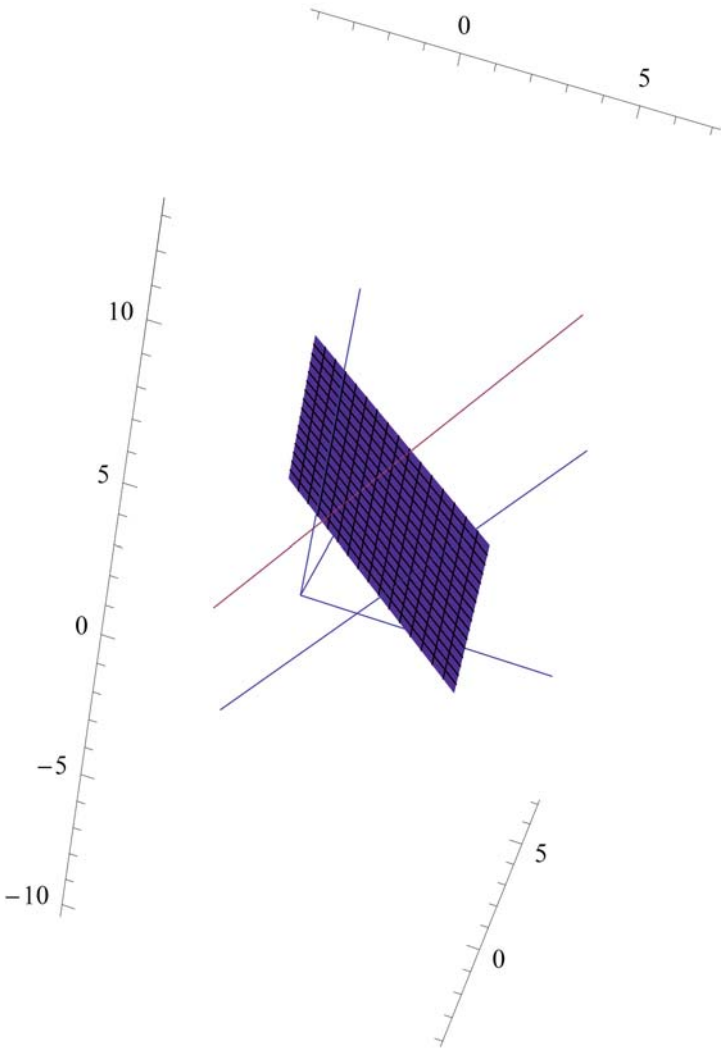
```
ro = Det[{X - A, B - A, s}] == 0
```

$$-15 + 5x + 5y == 0$$

```
gro = ParametricPlot3D[{t, 3 - t, u}, {t, -2, 5}, {u, 0, 5}]
```




```
Show[gpq, gro, osx, osy, osz, Boxed → False]
```



- **Zistite vzájomnú polohu priamok**
  p: x = 1 + t, y = 2 - 2t, z = 4 - t,
  q: x = - r, y = 4 + r, z = 5 - 2r.
  Ak ležia v jednej rovine, napíšte rovnicu roviny, ktorá je nimi určená.

```
Clear[t, r]
```

```
p = {1 + t, 2 - 2 t, 4 - t};
```

```
q = {-r, 4 + r, 5 - 2 r};
```

```
Solve[p == q, {t, r}]
```

```
{{t → -1, r → 0}}
```

```
R = q /. r → 0
```

```
{0, 4, 5}
```

```
sp = {1, -2, -1}; sq = {-1, 1, -2};
```

```
X = {x, y, z};
```

```
ro = Det[{X - R, sp, sq}]
```

```
-7 + 5 x + 3 y - z
```

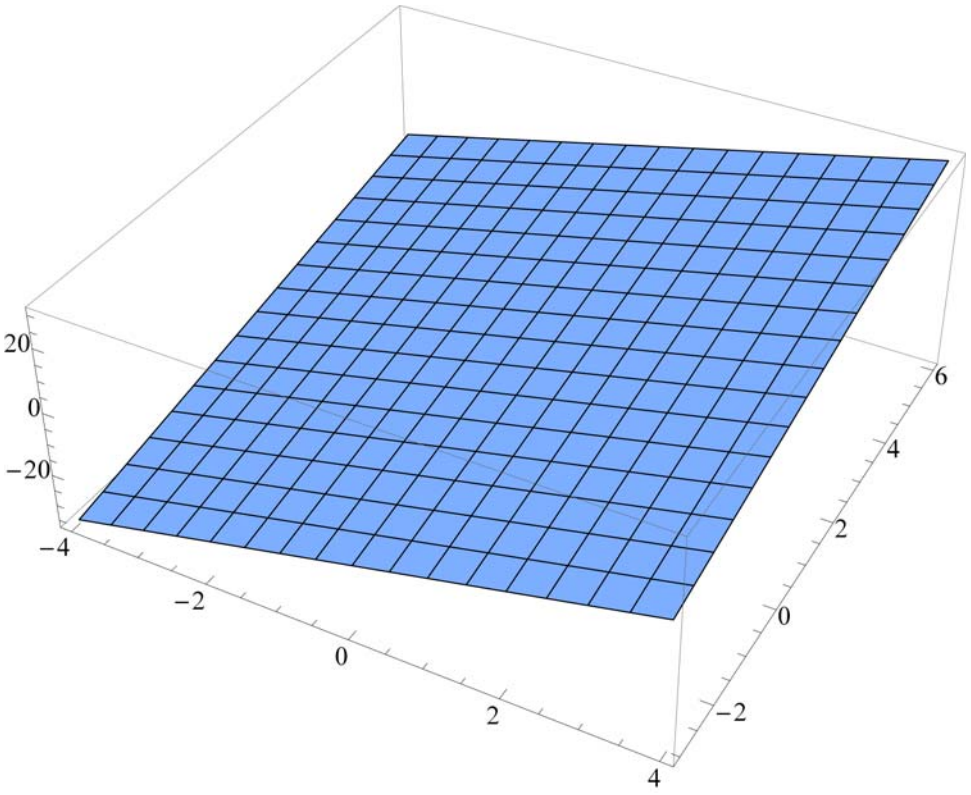
```
Solve[ro == 0, z]
```

```
{{z → -7 + 5 x + 3 y}}
```

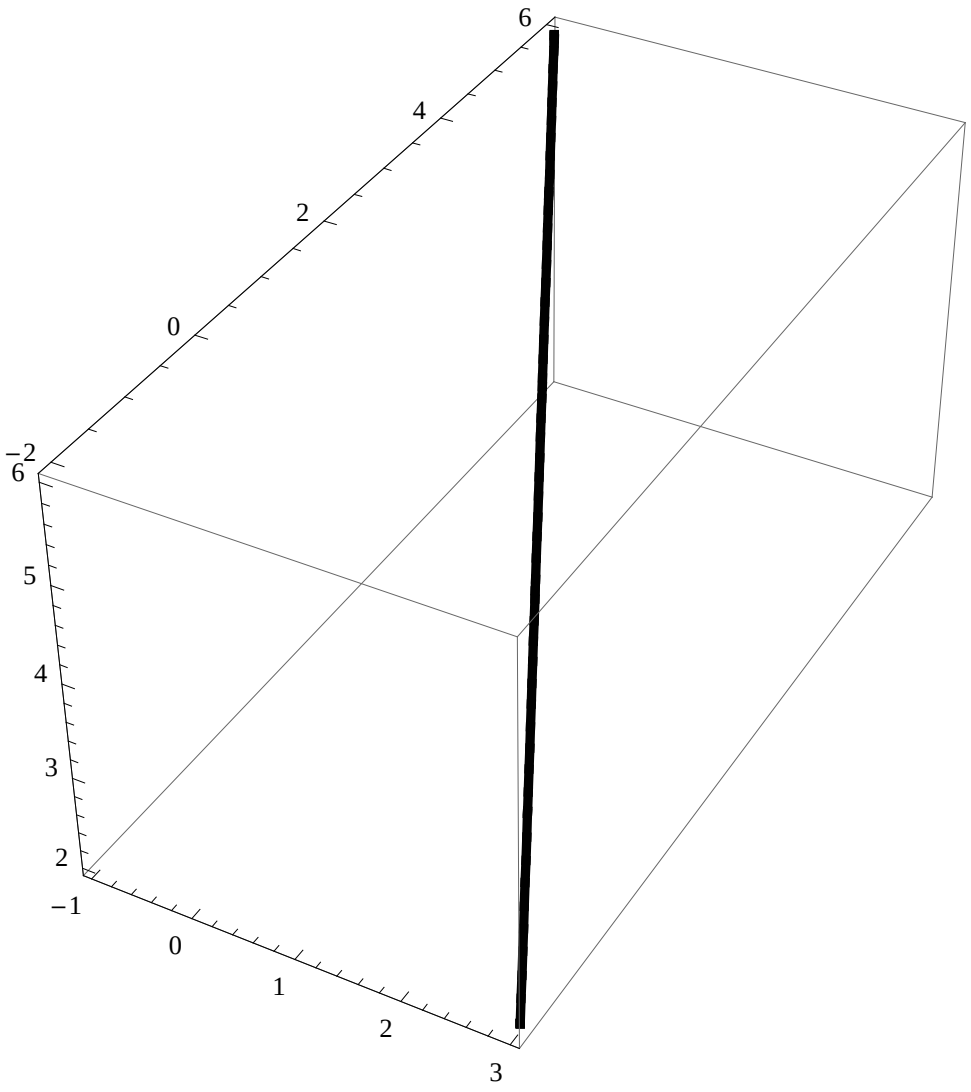
```
zsur = z /. %[[1]]
```

```
-7 + 5 x + 3 y
```

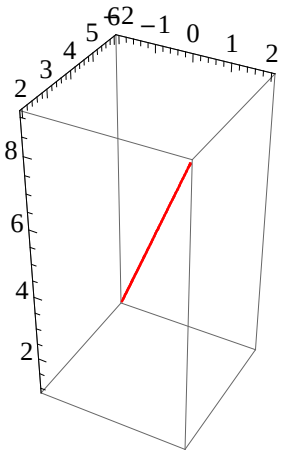
```
gro = Plot3D[zsur, {x, -4, 4}, {y, -3, 6}]
```



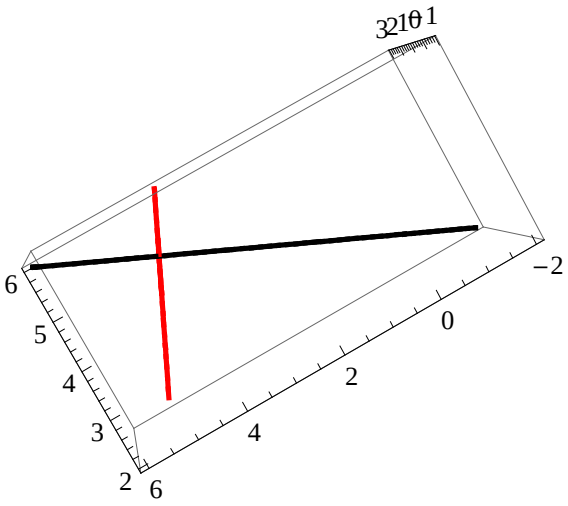

```
gp = ParametricPlot3D[p, {t, -2, 2}, PlotStyle -> Thickness[0.01]]
```



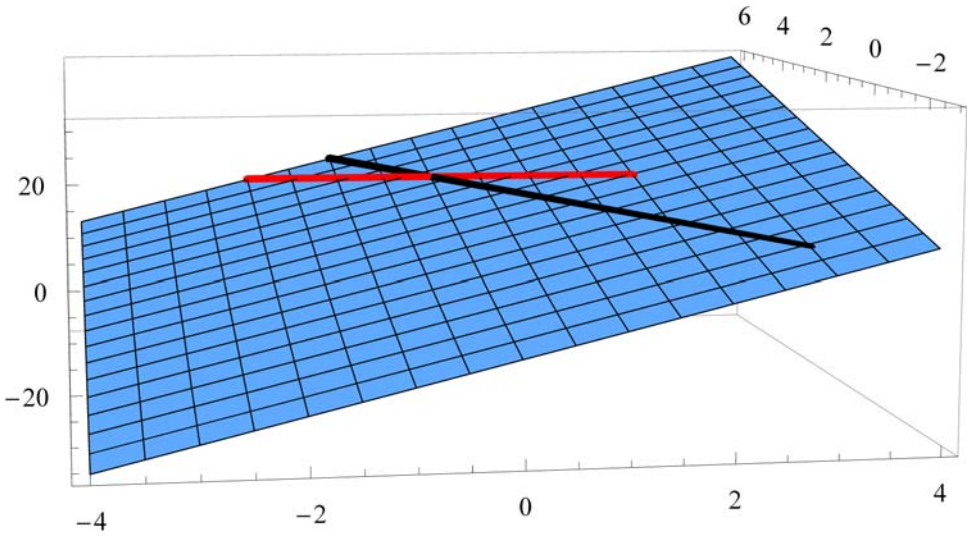
```
gq = ParametricPlot3D[q, {r, -2, 2},
  PlotStyle → {Thickness[0.01], RGBColor[1, 0, 0]}]
```




```
Show[gp, gq]
```



```
Show[gro, gp, gq]
```



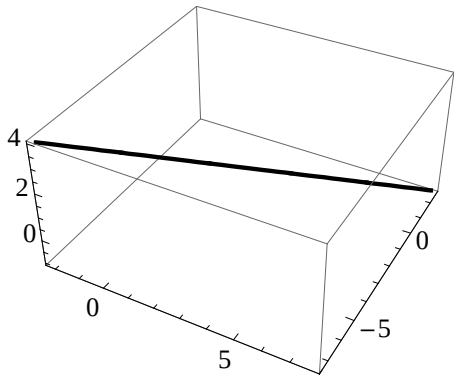
- **Ukážte, že roviny**
  ρ: x + 2y + 6z - 7 = 0,
  δ: 3x + y +8z - 18 = 0
  sú rôznobežné a napíšte parametrické rovnice priamky, ktorá je nimi určená.

```
ro = x + 2 y + 6 z - 7 == 0;
sigma = 3 x + y + 8 z - 18 == 0;

Solve[{ro, sigma}, {x, y, z}]
```

```
Solve::svars : Equations may not give solutions for all "solve" varia
```

$$\left\{\left\{x \to \frac{29}{5} - 2\,z,\ y \to \frac{3}{5} - 2\,z\right\}\right\}$$

```
p = {29 / 5 - 2 t, 3 / 5 - 2 t, t};

gp = ParametricPlot3D[p, {t, -1, 4}, PlotStyle → Thickness[0.01]]
```



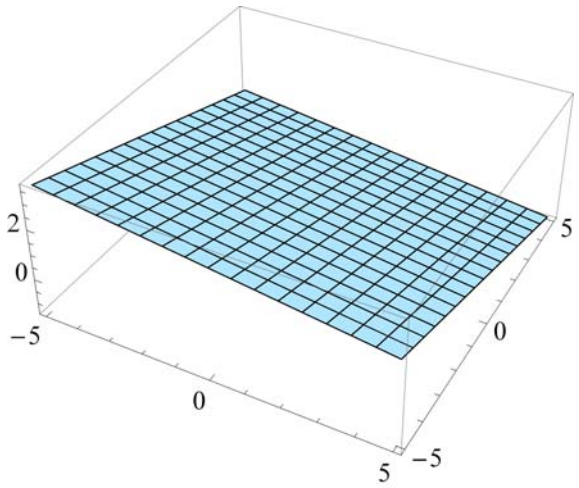

```
rov1 = Solve[ro, z]
```

$$\left\{\left\{z \to \frac{1}{6}\,(7 - x - 2\,y)\right\}\right\}$$

```
zsur1 = z /. rov1[[1]]
```

$$\frac{1}{6}\,(7 - x - 2\,y)$$

```
gro = Plot3D[zsur1, {x, -5, 5}, {y, -5, 5}]
```



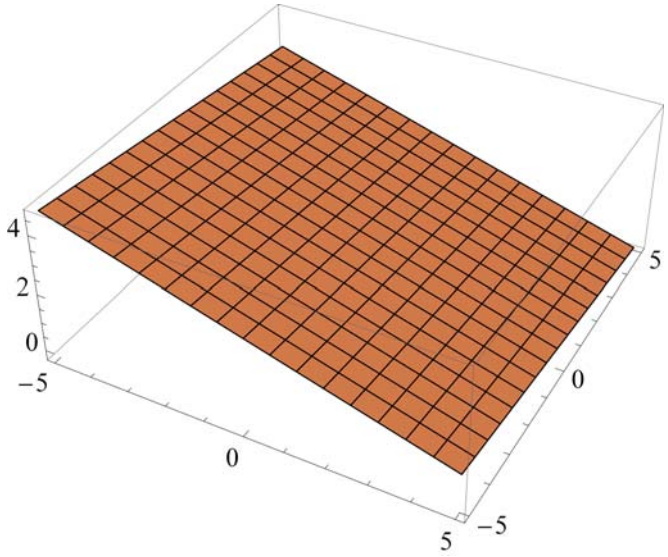
```
rov2 = Solve[sigma, z]
```

$$\left\{\left\{z \to \frac{1}{8}\,(18 - 3\,x - y)\right\}\right\}$$

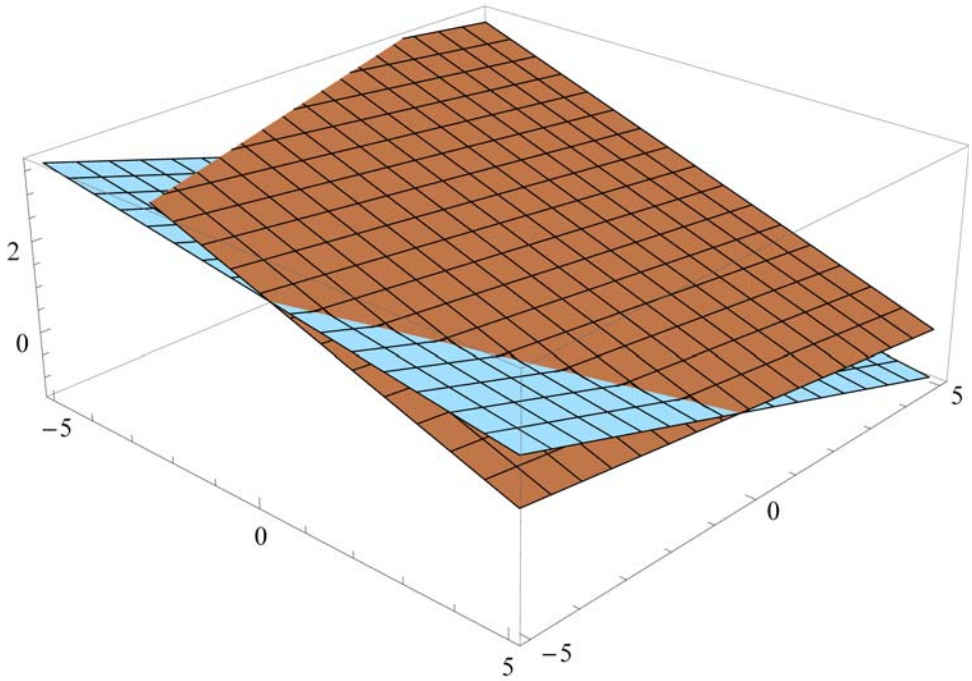
```
zsur2 = z /. rov2[[1]]
```

$$\frac{1}{8}\,(18 - 3\,x - y)$$

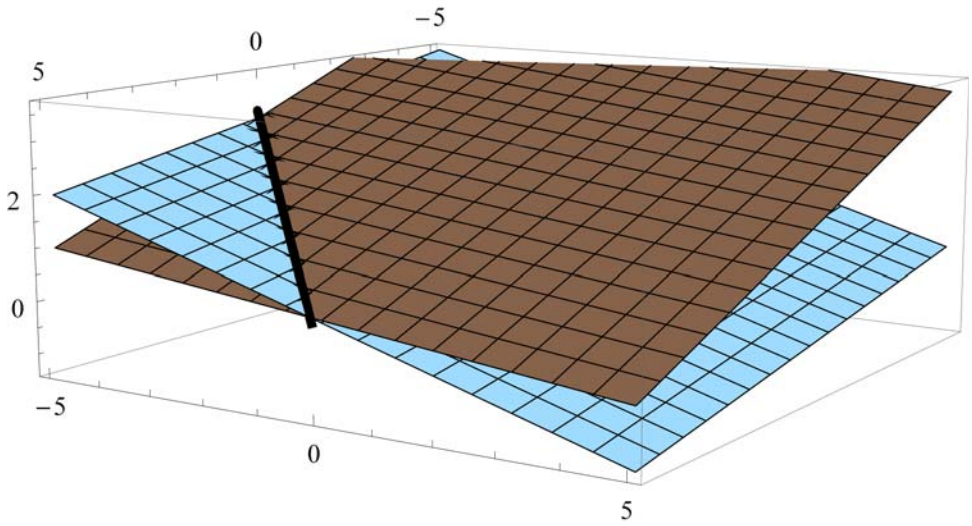
```
gsigma = Plot3D[zsur2, {x, -5, 5}, {y, -5, 5},
  PlotStyle -> RGBColor[1, 0.501961, 0.25098]]
```




```
Show[gro, gsigma]
```



```
Show[gro, gsigma, gp]
```



- **Zistite vzájomnú polohu troch rovín**
  α: 3x + y + z - 12 = 0,
  β: 2x + 3y + z - 11 = 0,
  γ: x - 2y + z - 3 = 0.

```
alfa = 3 x + y + z - 12 == 0;
beta = 2 x + 3 y + z - 11 == 0;
gama = x - 2 y + z - 3 == 0;

Solve[{alfa, beta, gama}, {x, y, z}]
```

```
{{x → 3, y → 1, z → 2}}
```

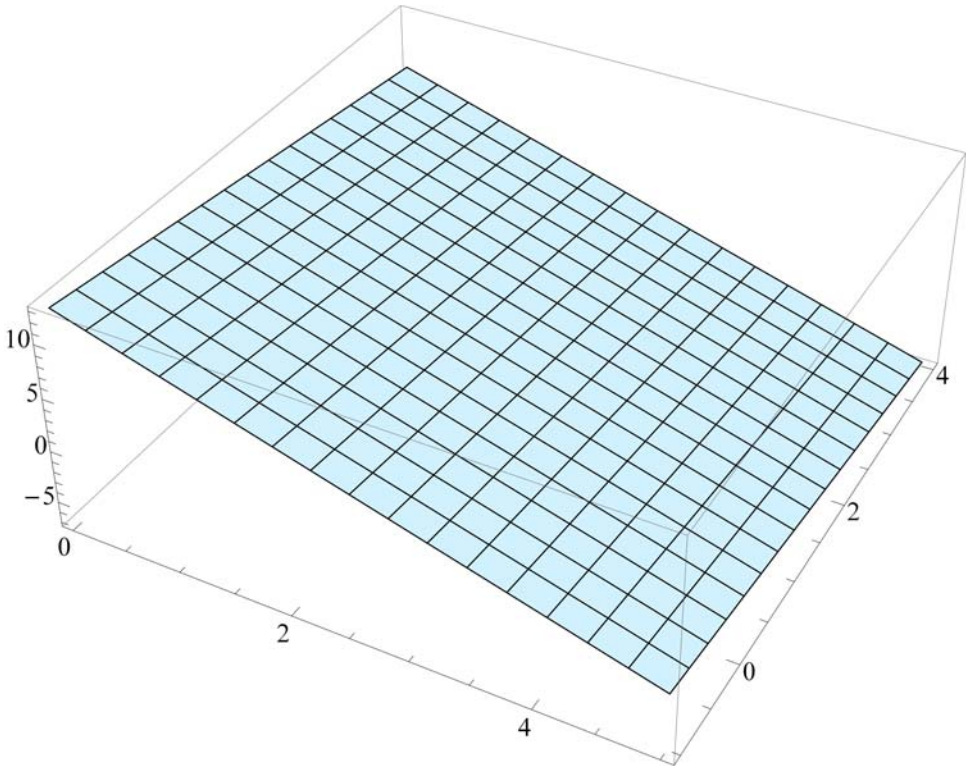
```
r1 = Solve[alfa, z]
```

```
{{z → 12 - 3 x - y}}
```

```
zsur1 = z /. r1[[1]]
```

```
12 - 3 x - y
```

```
galfa = Plot3D[zsur1, {x, 0, 5}, {y, -1, 4}]
```



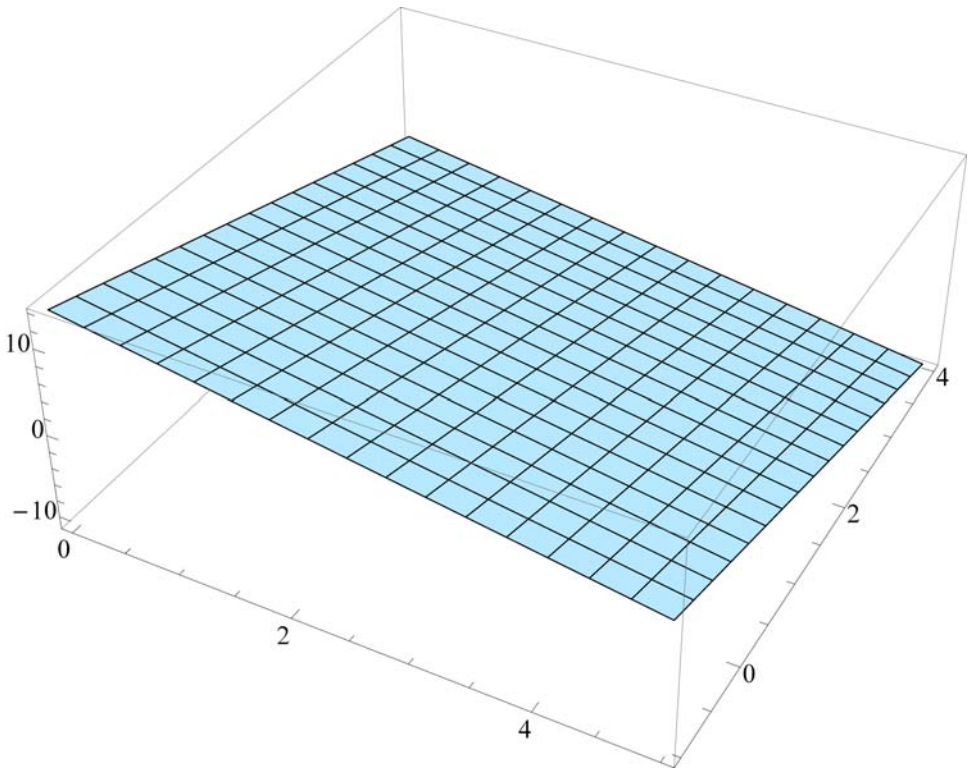

```
r2 = Solve[beta, z]
```

```
{{z → 11 − 2 x − 3 y}}
```

```
zsur2 = z /. r2[[1]]
```

```
11 − 2 x − 3 y
```

```
gbeta = Plot3D[zsur2, {x, 0, 5}, {y, -1, 4}]
```



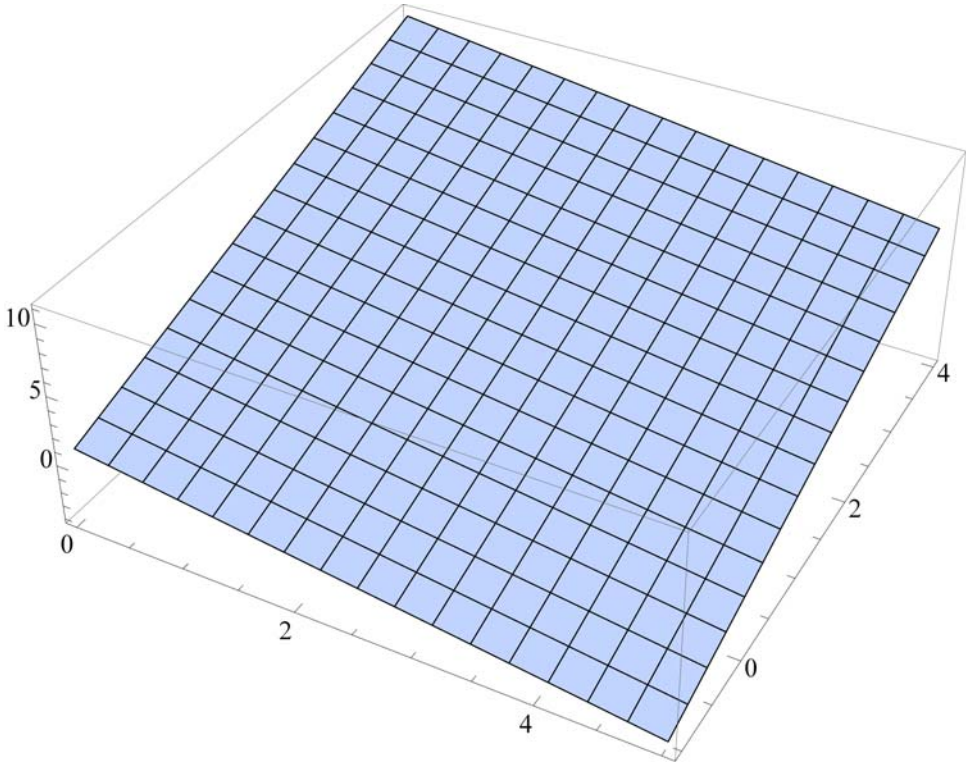

```
r3 = Solve[gama, z]
```

```
{{z → 3 − x + 2 y}}
```

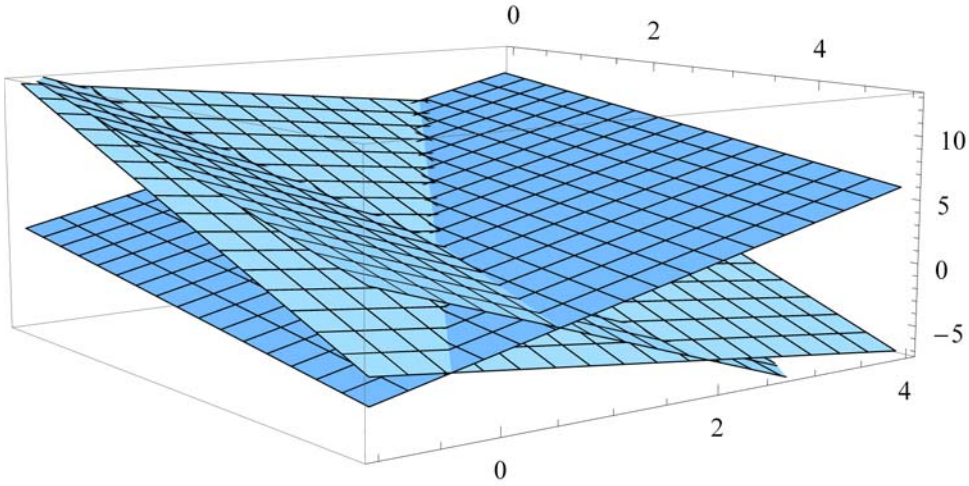
```
zsur3 = z /. r3[[1]]
```

$3 - x + 2\,y$

```
ggama = Plot3D[zsur3, {x, 0, 5}, {y, -1, 4}]
```




```
Show[galfa, gbeta, ggama]
```




```
Clear[alfa, beta, gama]
```

- **Zistite vzájomnú polohu troch rovín**
  α: x + 2y + 3z- 10 = 0,
  β: 2x - y - z + 5 = 0,
  γ: 3x + y + 2z - 5 = 0.

```
alfa = x + 2 y + 3 z - 10 == 0;
beta = 2 x - y - z + 5 == 0;
gama = 3 x + y + 2 z - 5 == 0;

Solve[{alfa, beta, gama}, {x, y, z}]
```

$$\left\{\left\{x \to -\frac{z}{5},\ y \to 5 - \frac{7\,z}{5}\right\}\right\}$$

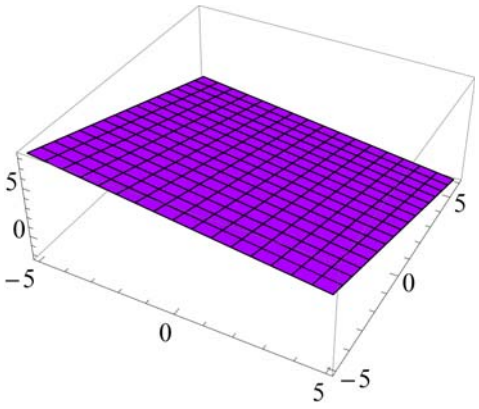
```
r1 = Solve[alfa, z]
```

$$\left\{\left\{z \to \frac{1}{3}\,(10 - x - 2\,y)\right\}\right\}$$

```
zsur1 = z /. r1[[1]]
```

$$\frac{1}{3}\,(10 - x - 2\,y)$$

```
galfa = Plot3D[zsur1, {x, -5, 5}, {y, -5, 6},
  PlotStyle -> RGBColor[1, 0, 1]]
```



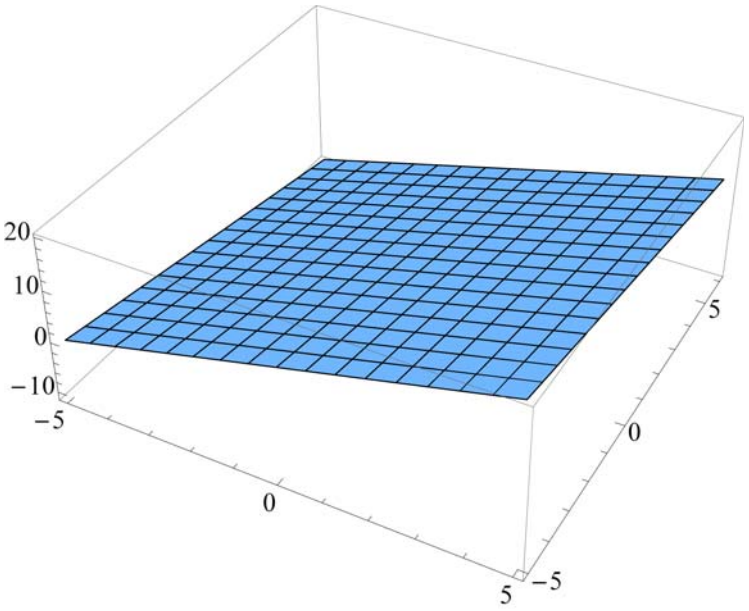

```
r2 = Solve[beta, z]
```

$$\{\{z \to 5 + 2\,x - y\}\}$$

```
zsur2 = z /. r2[[1]]
```

$$5 + 2\,x - y$$

```
gbeta = Plot3D[zsur2, {x, -5, 5}, {y, -5, 6}]
```



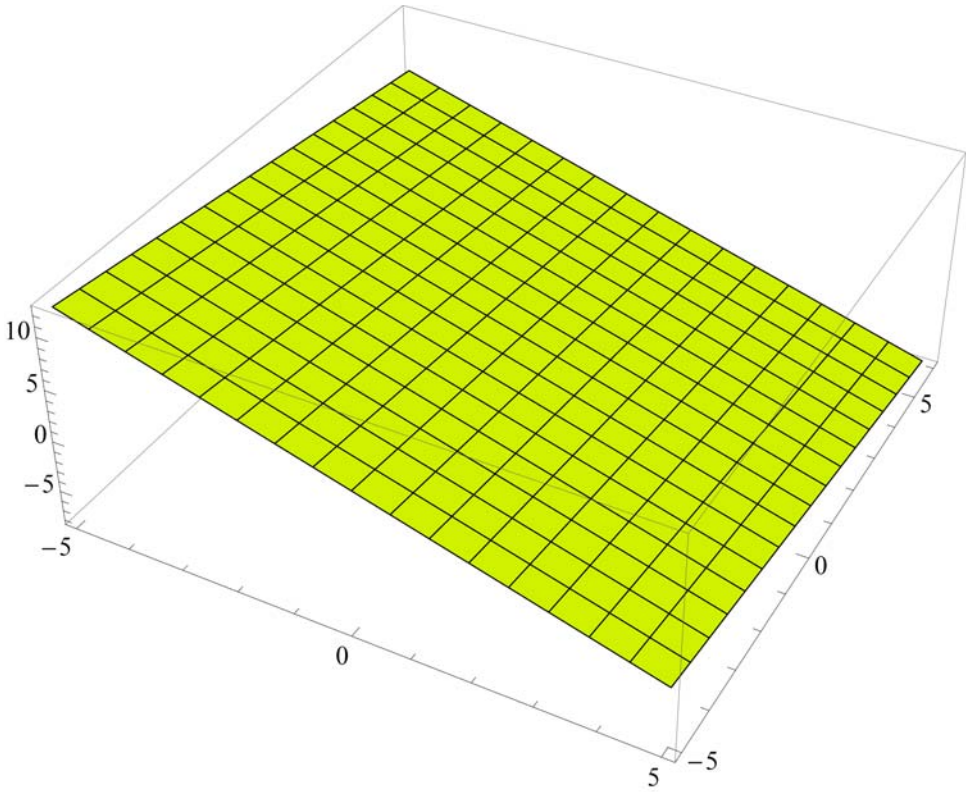

```
r3 = Solve[gama, z]
```

$$\left\{\left\{z \to \frac{1}{2}\,(5 - 3\,x - y)\right\}\right\}$$

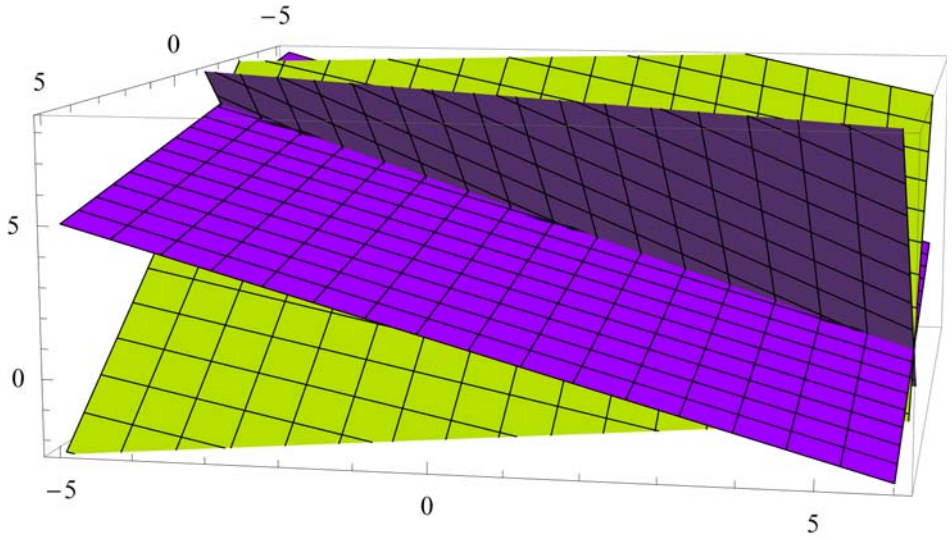
```
zsur3 = z /. r3[[1]]
```

$$\frac{1}{2}\,(5 - 3\,x - y)$$

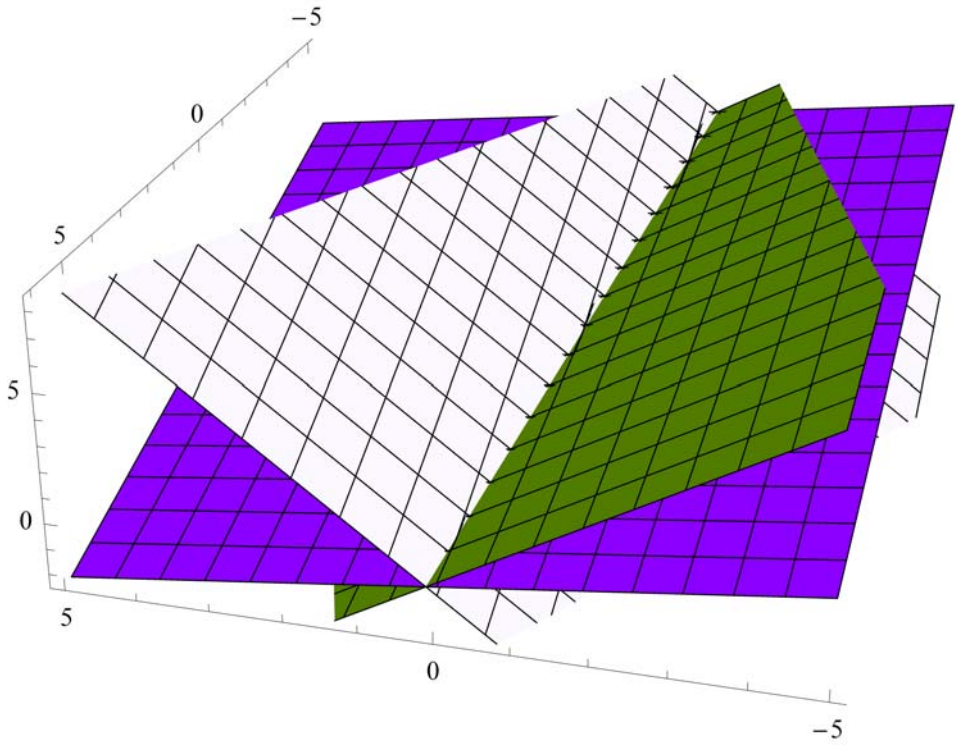
```
ggama = Plot3D[zsur3, {x, -5, 5}, {y, -5, 6},
  PlotStyle -> RGBColor[1, 1, 0]]
```



```
Show[galfa, gbeta, ggama]
```




```
Show[galfa, gbeta, ggama, Boxed → False]
```



- **Zistite vzájomnú polohu troch rovín**
  c) α:x + y + z - 6 = 0,
  β: 2x + y + 3z - 18 = 0,
  γ: 3x + 2y + 4z - 12= 0.

```
alfa = x + y + z - 6 == 0;
beta = 2 x + y + 3 z - 20 == 0;
gama = 3 x + 2 y + 4 z - 12 == 0;

Solve[{alfa, beta, gama}, {x, y, z}]
```

```
{}
```

- **Napíšte parametrické rovnice priamky p, ktorá prechádza bodom A=[2,1,0] a je kolmá na rovinu ρ: x + 2y + 2z - 12 = 0. Nájdite priesečník priamky p a roviny ρ.**

```
A = {2, 1, 0}; s = {1, 2, 2};
```

```
p = A + t * s
```

$$\{2 + t,\ 1 + 2\,t,\ 2\,t\}$$

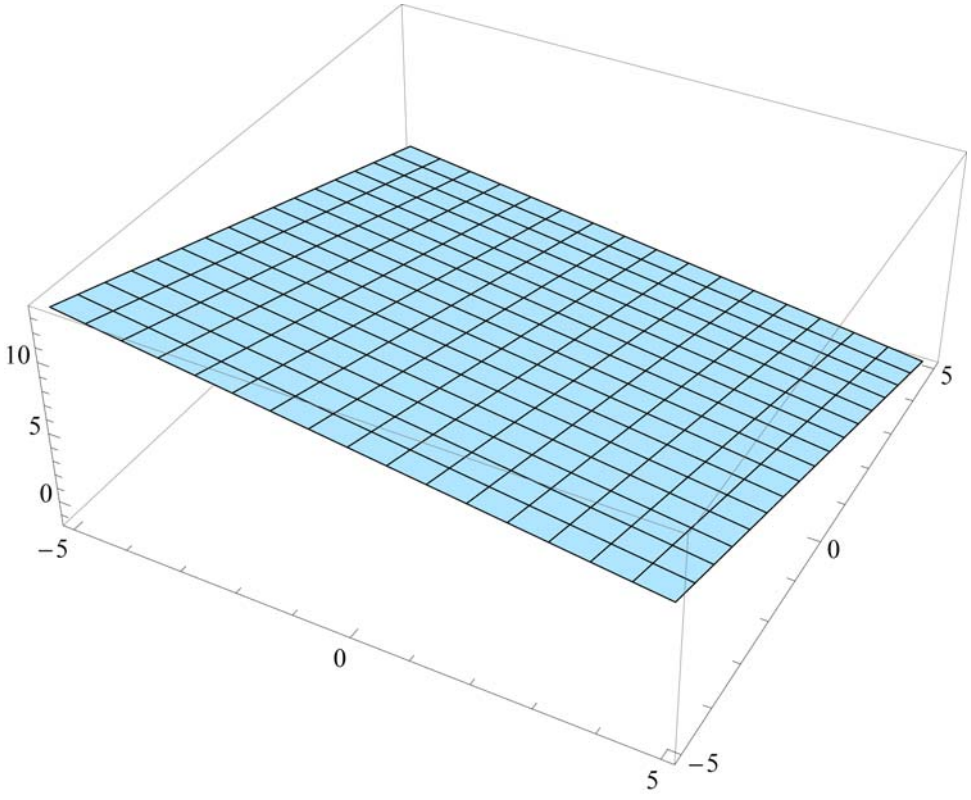
```
Solve[2 + t + 2 * (1 + 2 t) + 2 * 2 t - 12 == 0, t]
```

$$\left\{\left\{t \to \frac{8}{9}\right\}\right\}$$

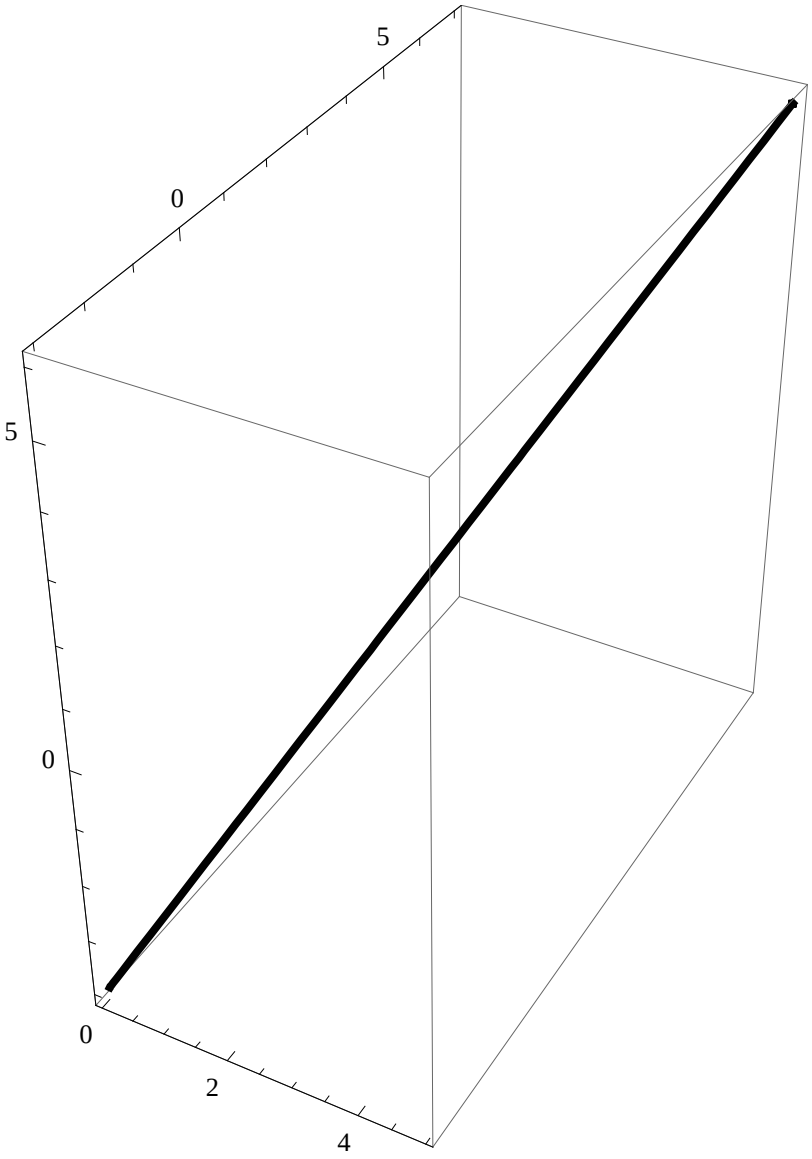
```
R = p /. t → 8 / 9
```

$$\left\{\frac{26}{9},\ \frac{25}{9},\ \frac{16}{9}\right\}$$

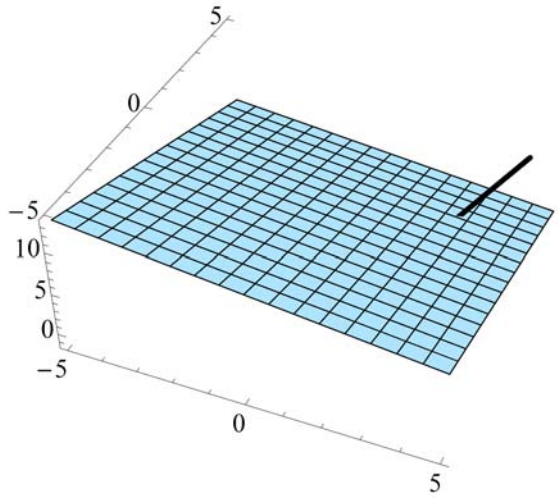
```
gro = Plot3D[6 - x / 2 - y, {x, -5, 5}, {y, -5, 5}]
```



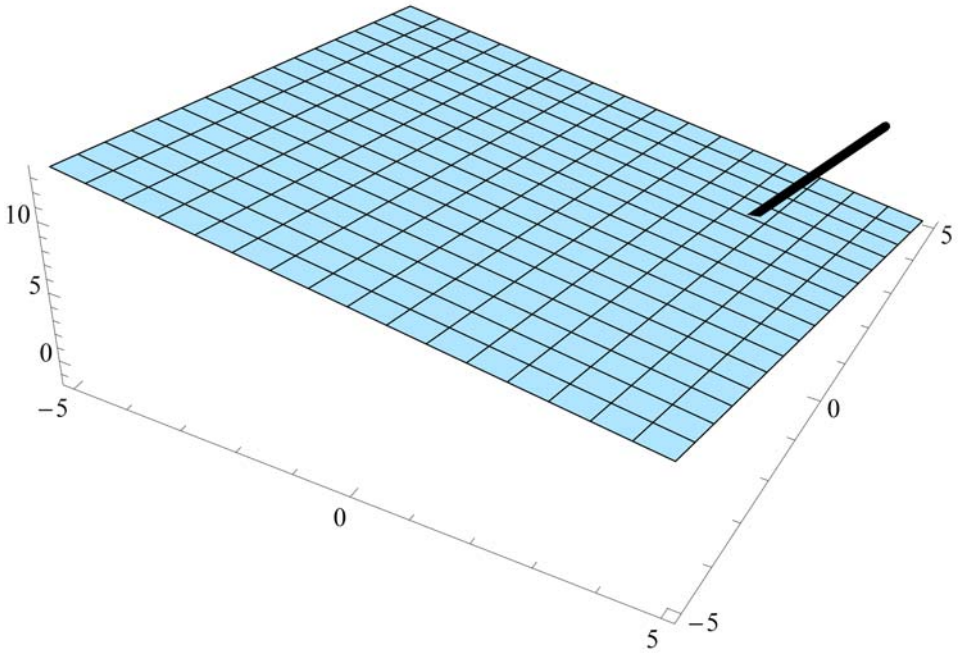
```
gp = ParametricPlot3D[p, {t, -2, 3}, PlotStyle → Thickness[0.01]]
```




```
Show[gro, gp, Boxed → False]
```



```
Show[gro, gp, Boxed → False]
```

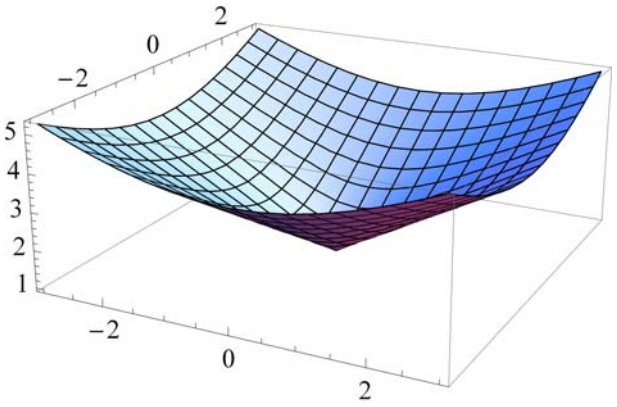


- **Nakreslite plochu určenú rovnicou** $z = 1 + \sqrt{x^2 + y^2}$

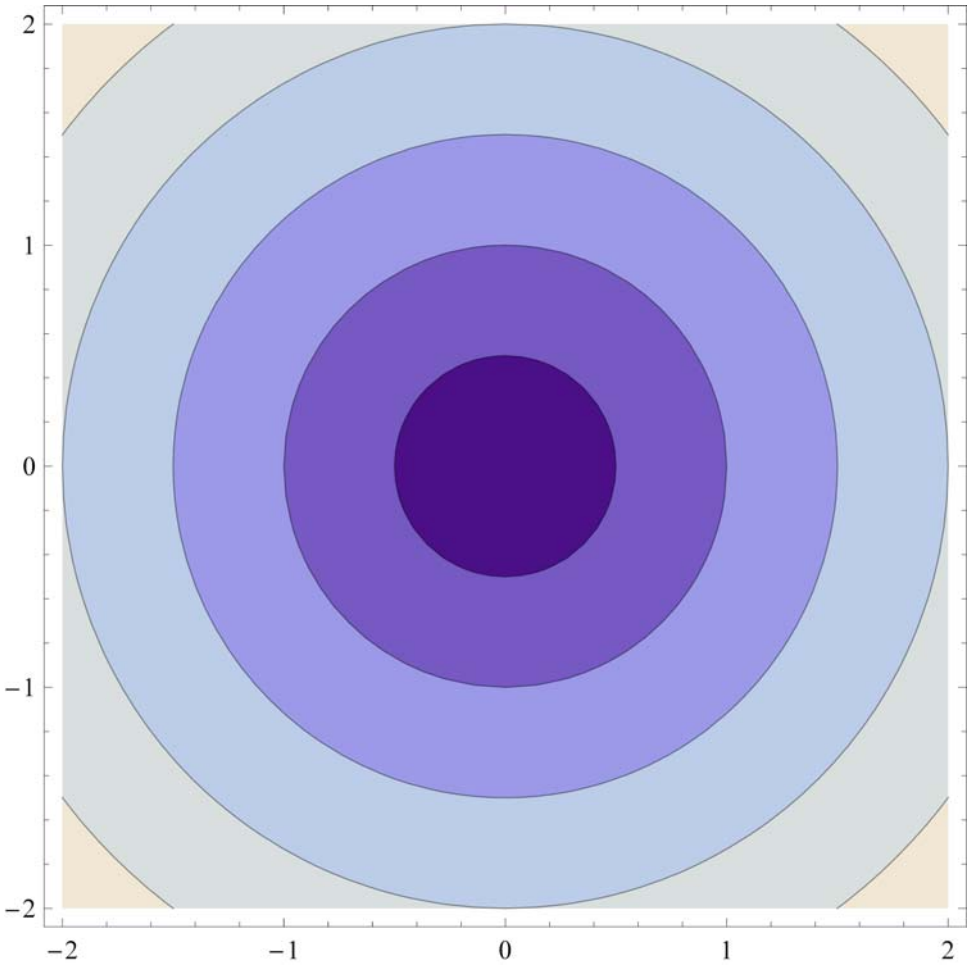
```
pl = 1 + Sqrt[x^2 + y^2]
```

$$1 + \sqrt{x^2 + y^2}$$

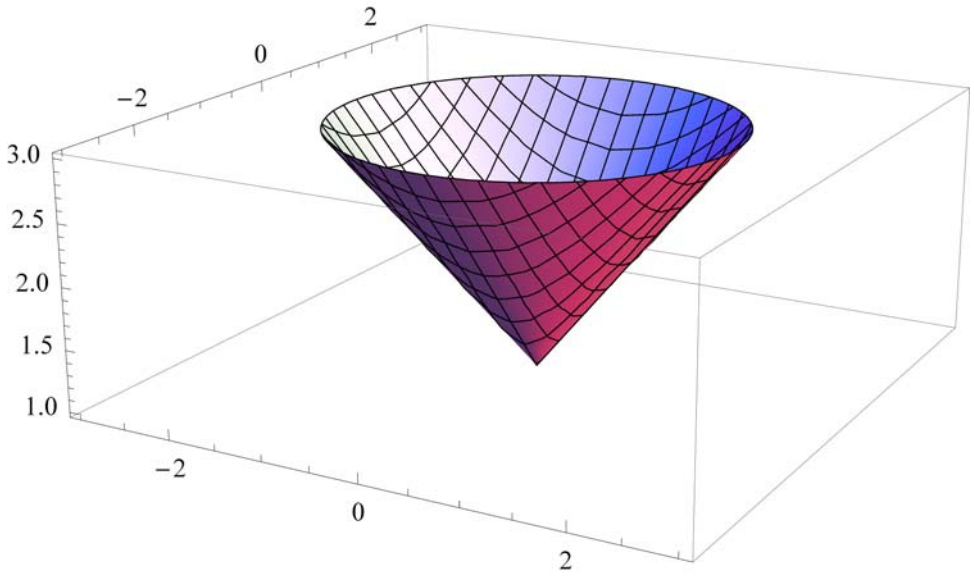
```
Plot3D[pl, {x, -3, 3}, {y, -3, 3}]
```



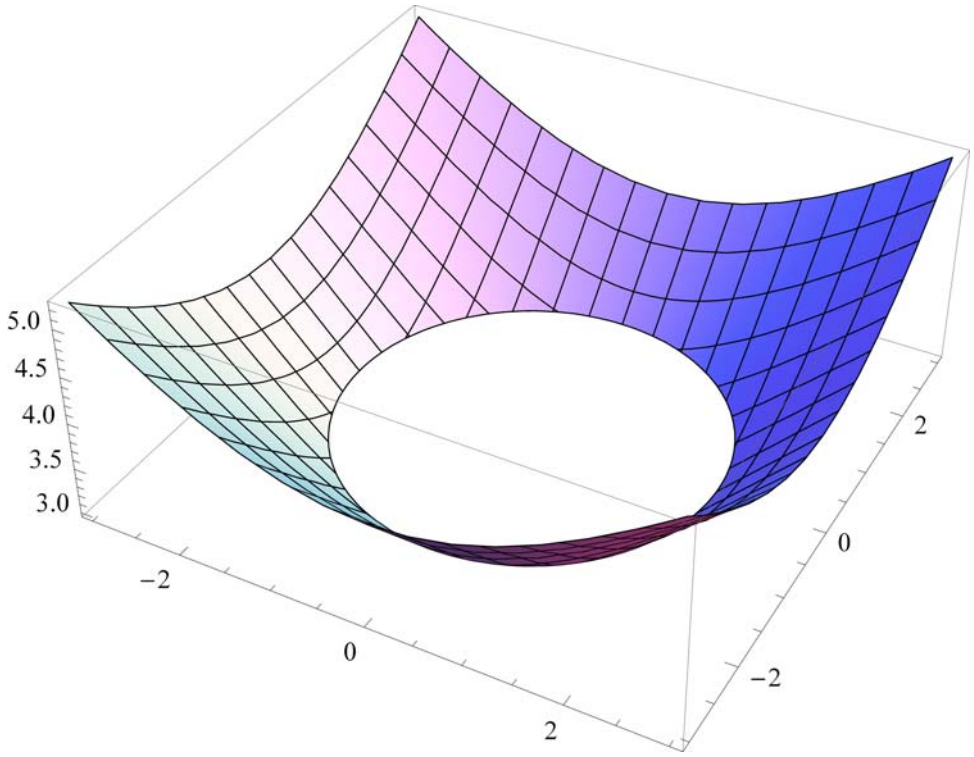

```
ContourPlot[pl, {x, -2, 2}, {y, -2, 2}]
```



```
Plot3D[pl, {x, -3, 3}, {y, -3, 3},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 ≤ 4]]
```




```
Plot3D[pl, {x, -3, 3}, {y, -3, 3},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 >= 4]]
```




```
Clear[pl]
```

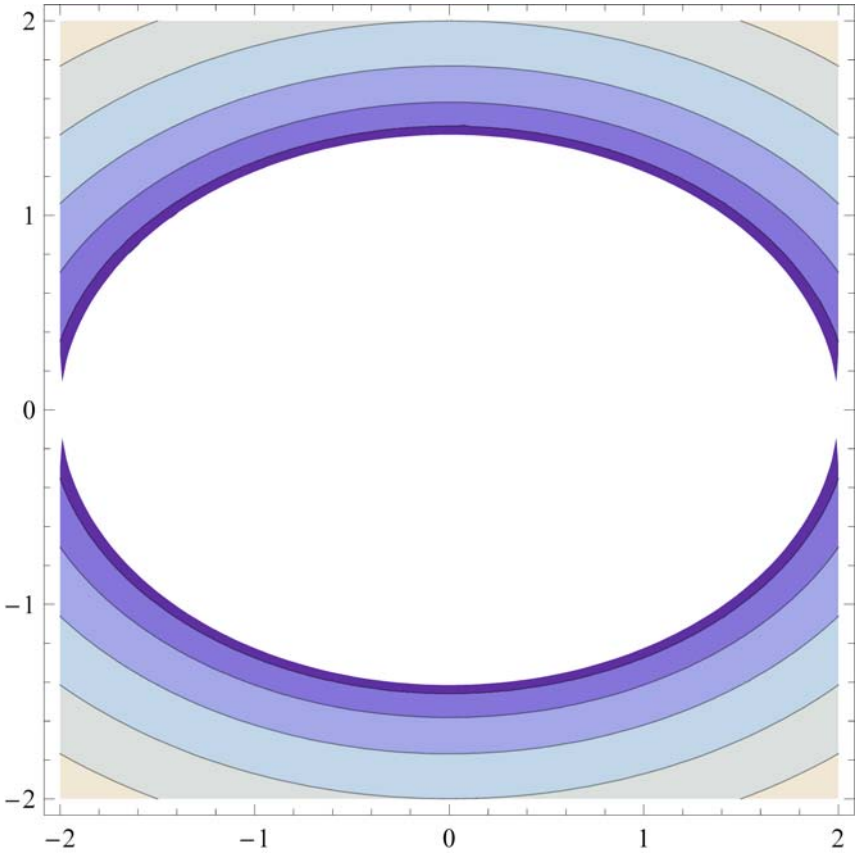
```
vrstvy := ContourPlot[pl, {x, -2, 2}, {y, -2, 2}]
```

- **Nakreslite plochu $z = \sqrt{x^2 + 2y^2 - 4}$**

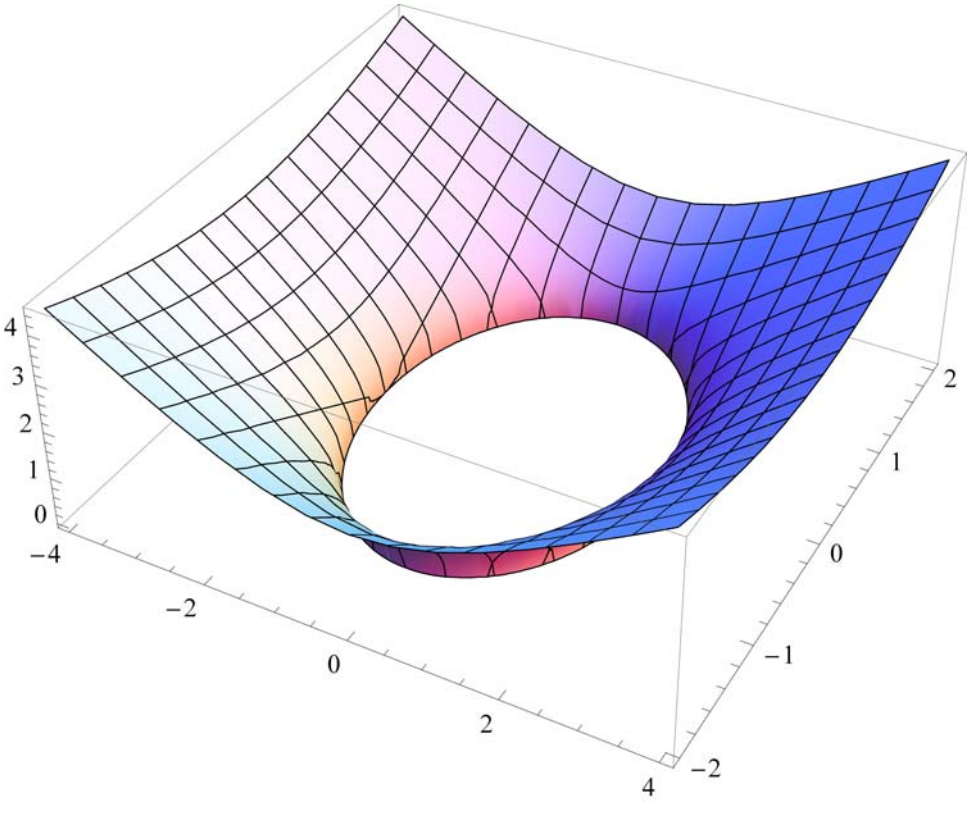
```
pl = Sqrt[x^2 + 2 * y^2 - 4]
```

$$\sqrt{-4 + x^2 + 2\,y^2}$$

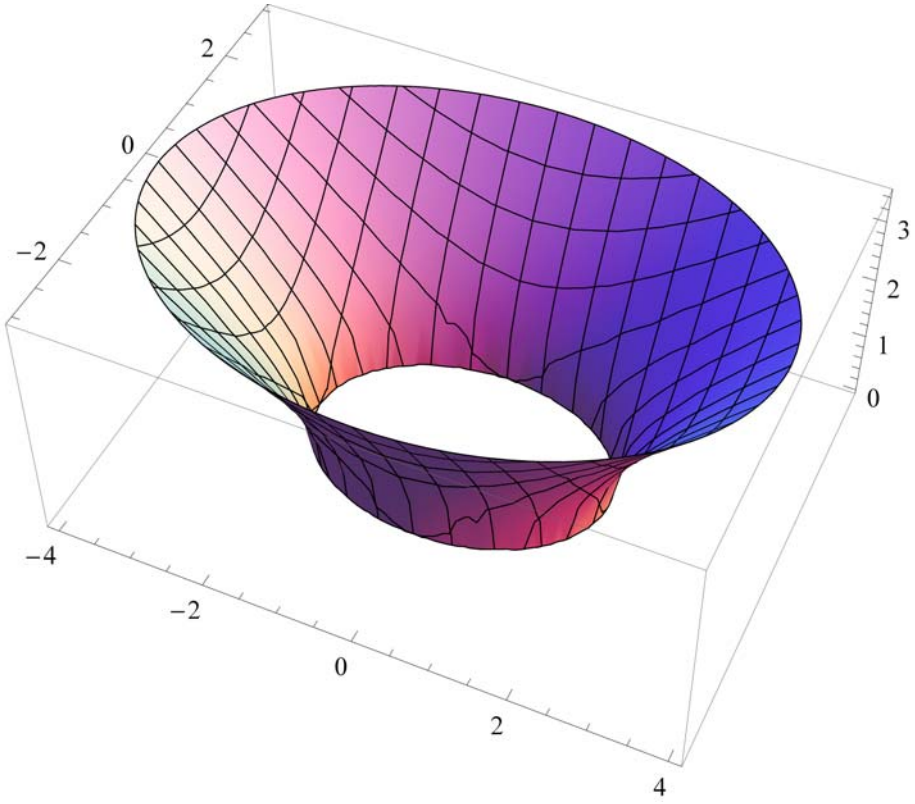
```
vrstvy
```




```
Plot3D[pl, {x, -4, 4}, {y, -2, 2},
 RegionFunction → Function[{x, y, z}, x^2 + 2 y^2 ≥ 4]]
```



```
Plot3D[pl, {x, -4, 4}, {y, -3, 3},
 RegionFunction → Function[{x, y, z}, x^2 + 2 y^2 ≤ 16],
 BoxRatios → Automatic]
```



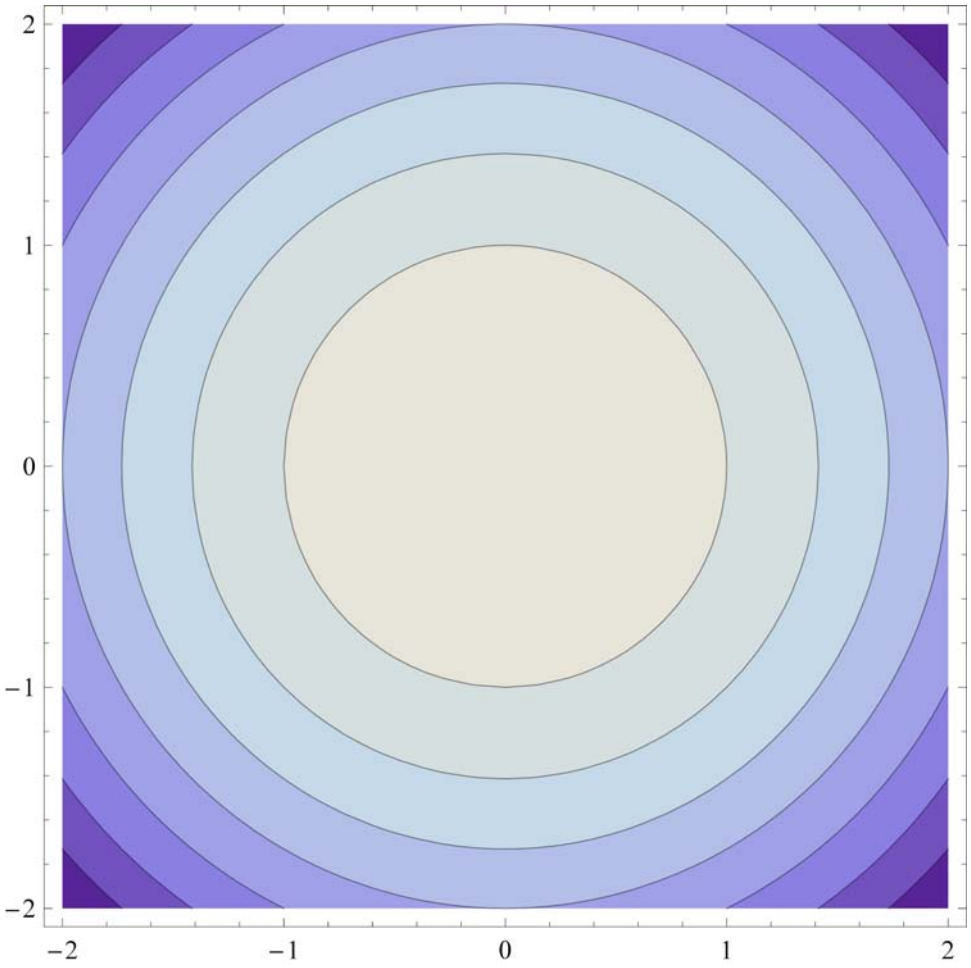

```
gpl := Plot3D[pl, {x, -3, 3}, {y, -3, 3}]
```

- **Nakreslite kvadratickú plochu $z = 4 - x^2 + y^2$.**

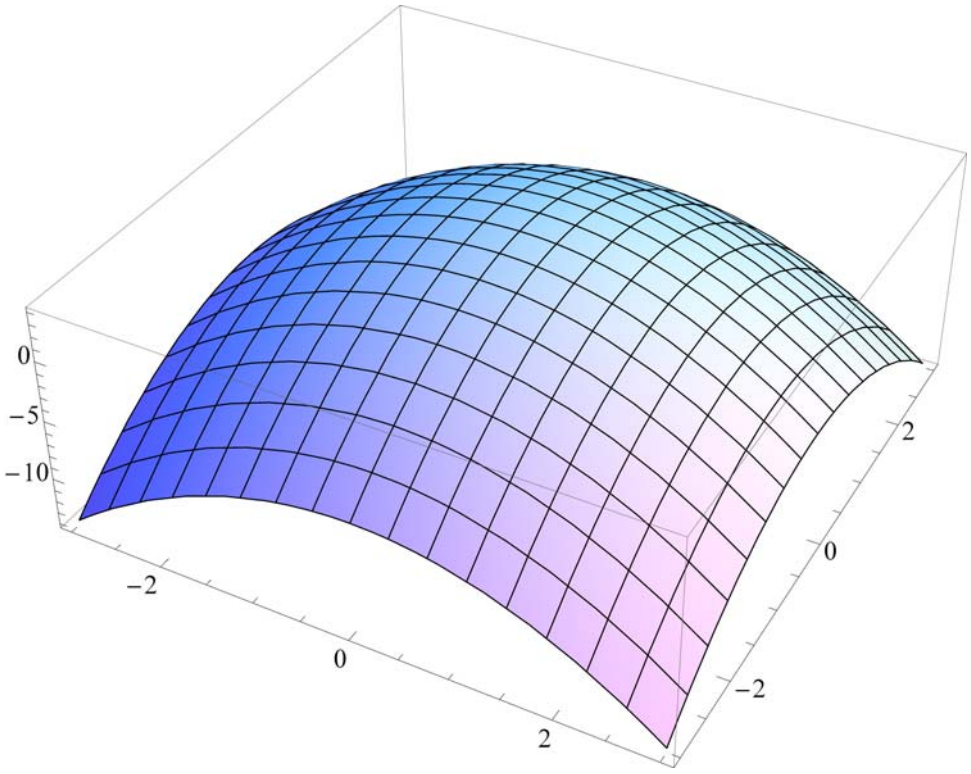
```
pl = 4 - x^2 - y^2
```

$4 - x^2 - y^2$

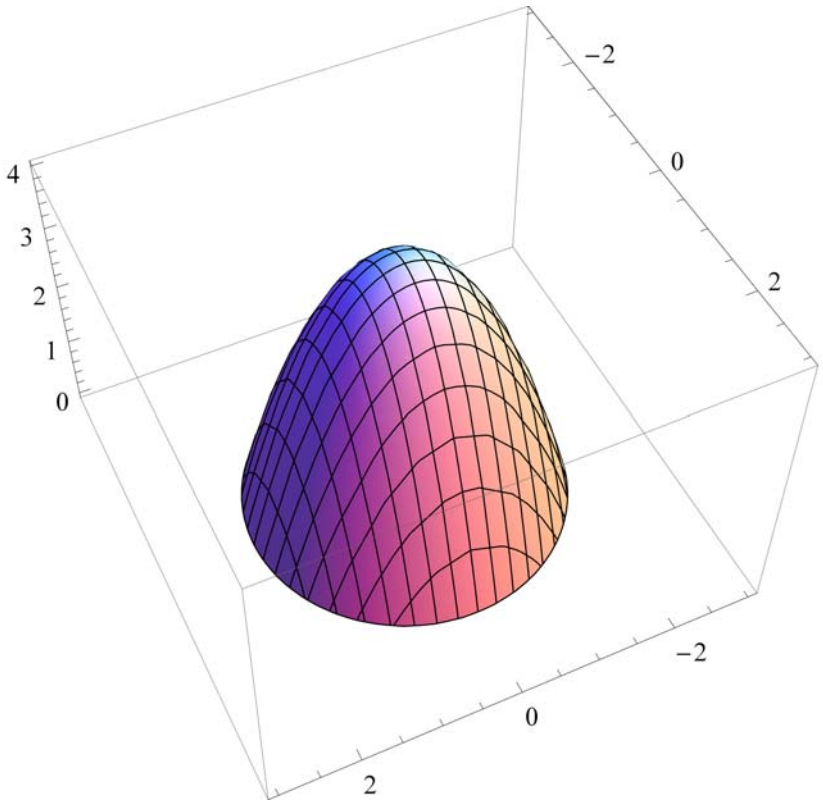
```
vrstvy
```




```
gpl
```



```
Plot3D[pl, {x, -3, 3}, {y, -3, 3},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 ≤ 4],
 BoxRatios → Automatic]
```

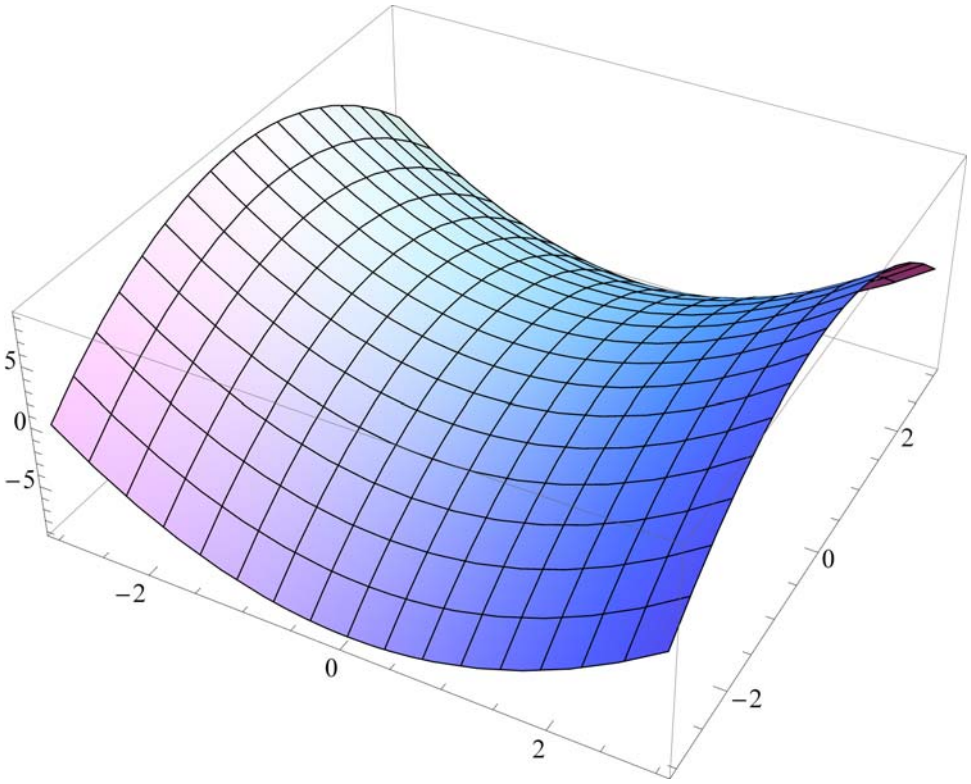


- **Nakreslite kvadratickú plochu $z = x^2 - y^2$.**

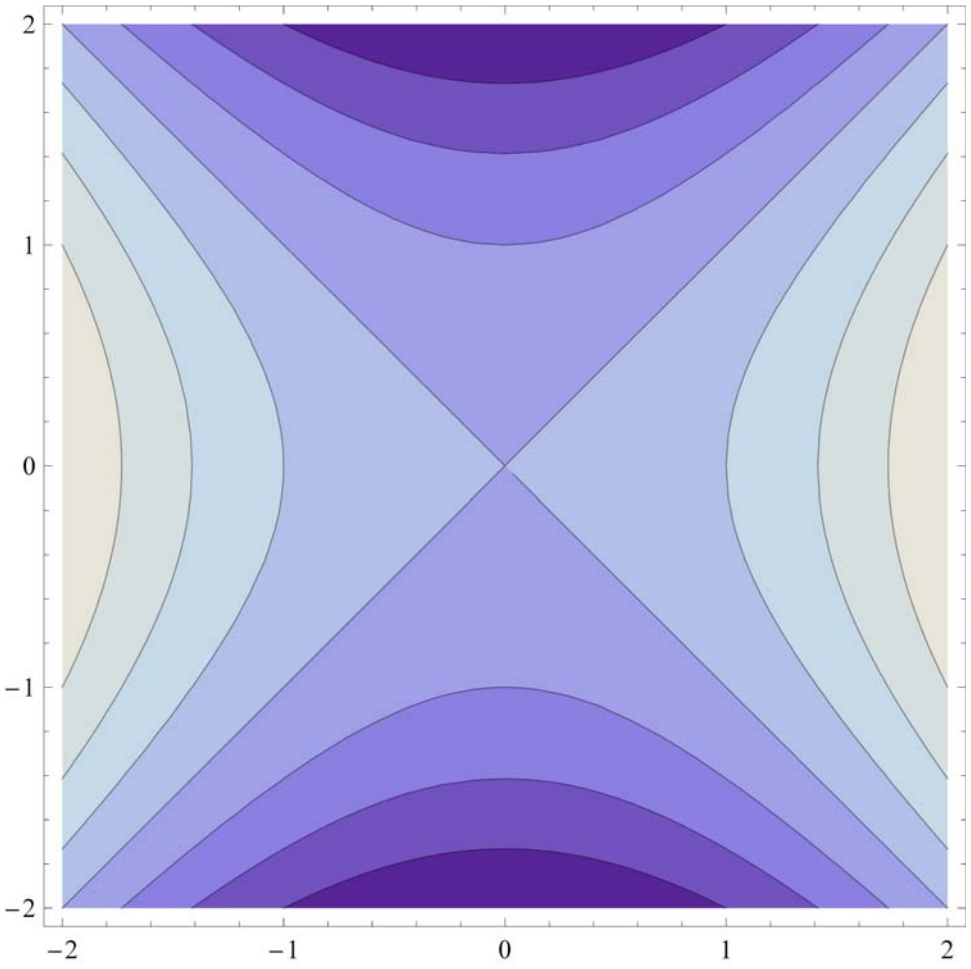
```
pl = x^2 - y^2
```

$x^2 - y^2$

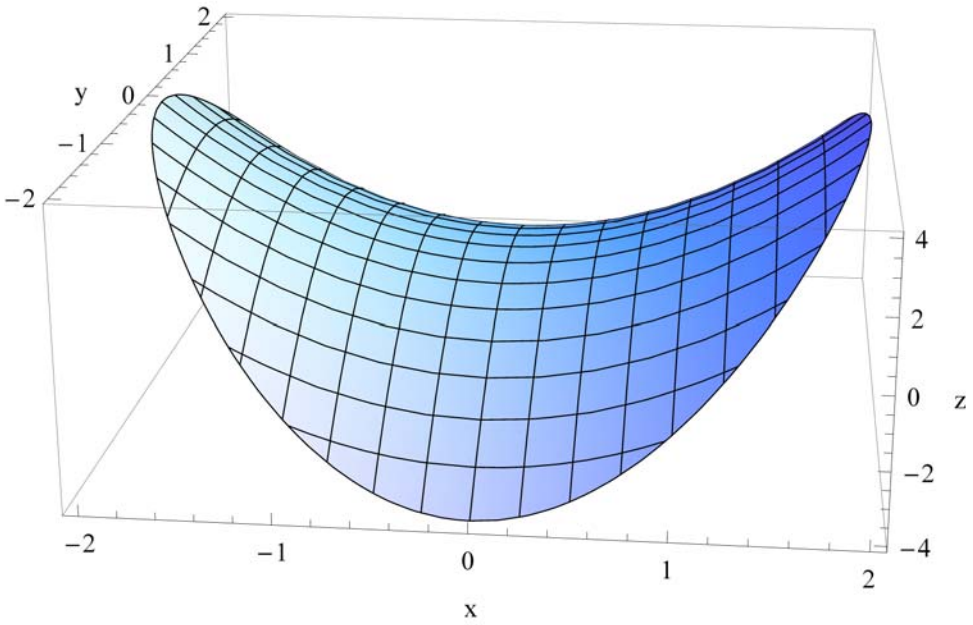
```
gpl
```



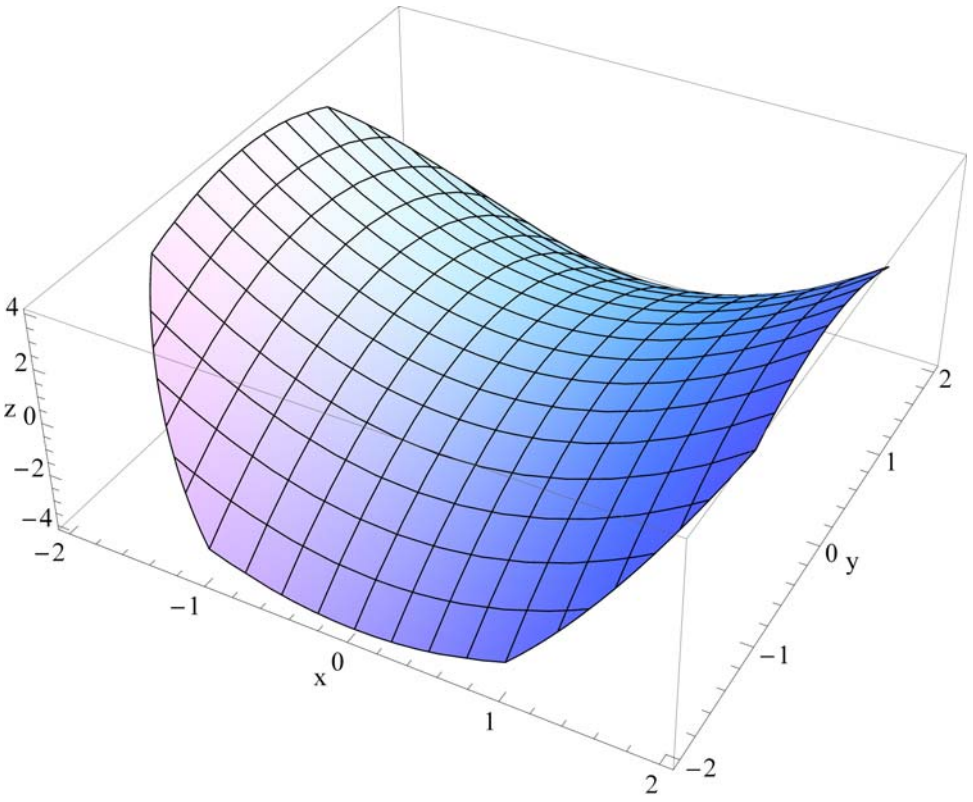

```
vrstvy
```



```
Plot3D[pl, {x, -2, 2}, {y, -2, 2},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 ≤ 4],
 AxesLabel → {"x", "y", "z"}]
```




```
Plot3D[pl, {x, -2, 2}, {y, -2, 2},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 ≤ 5],
 AxesLabel → {"x", "y", "z"}]
```

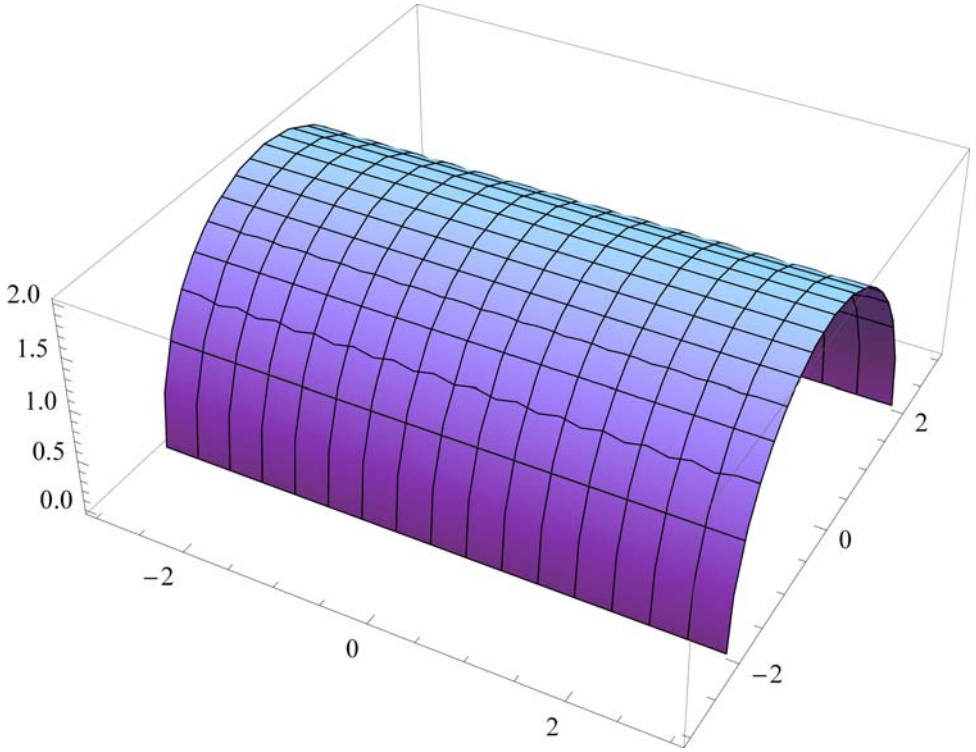


- **Nakreslite plochu $z = \sqrt{4 - y^2}$.**

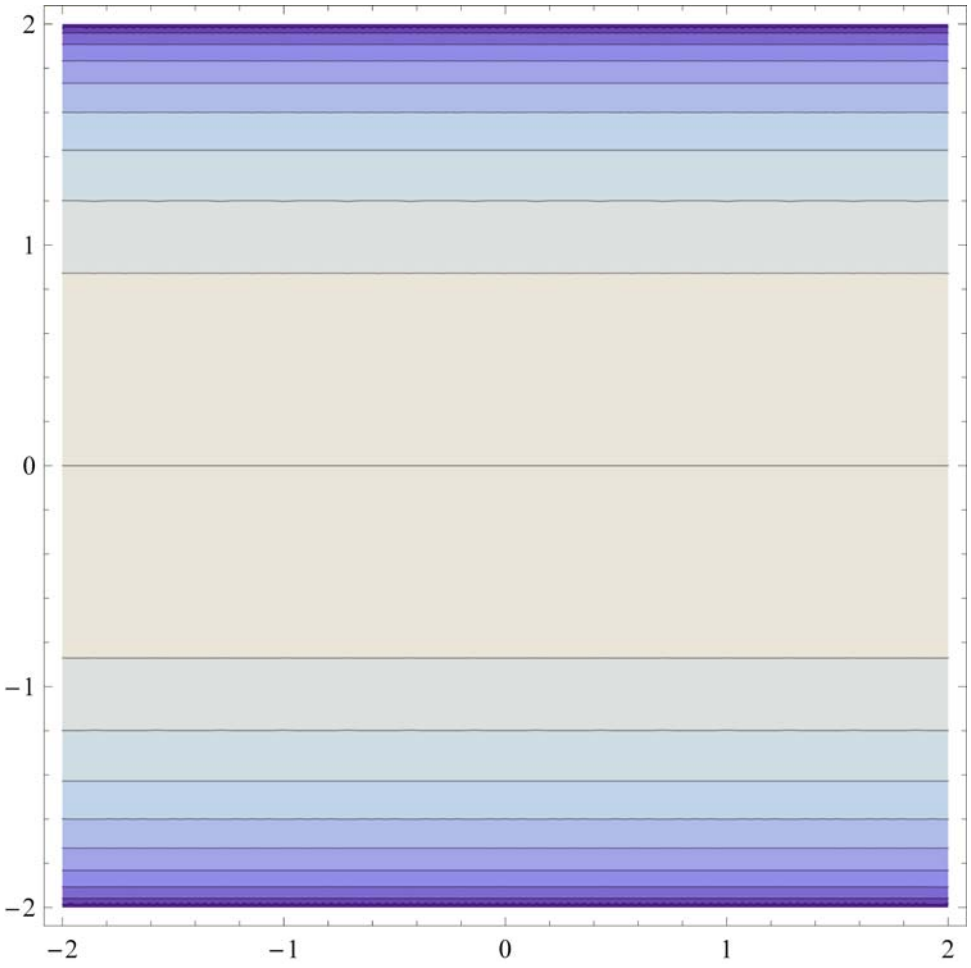
```
pl = Sqrt[4 - y^2]
```

$$\sqrt{4 - y^2}$$

```
gpl
```



vrstvy

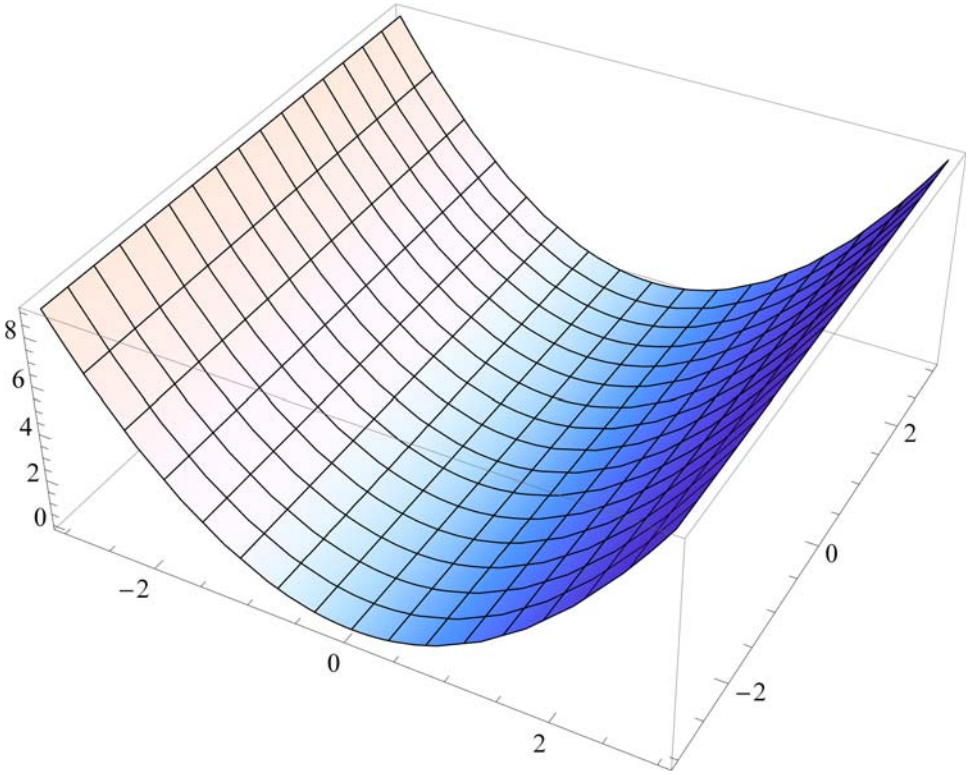


- **Nakreslite plochu $z = x^2$**

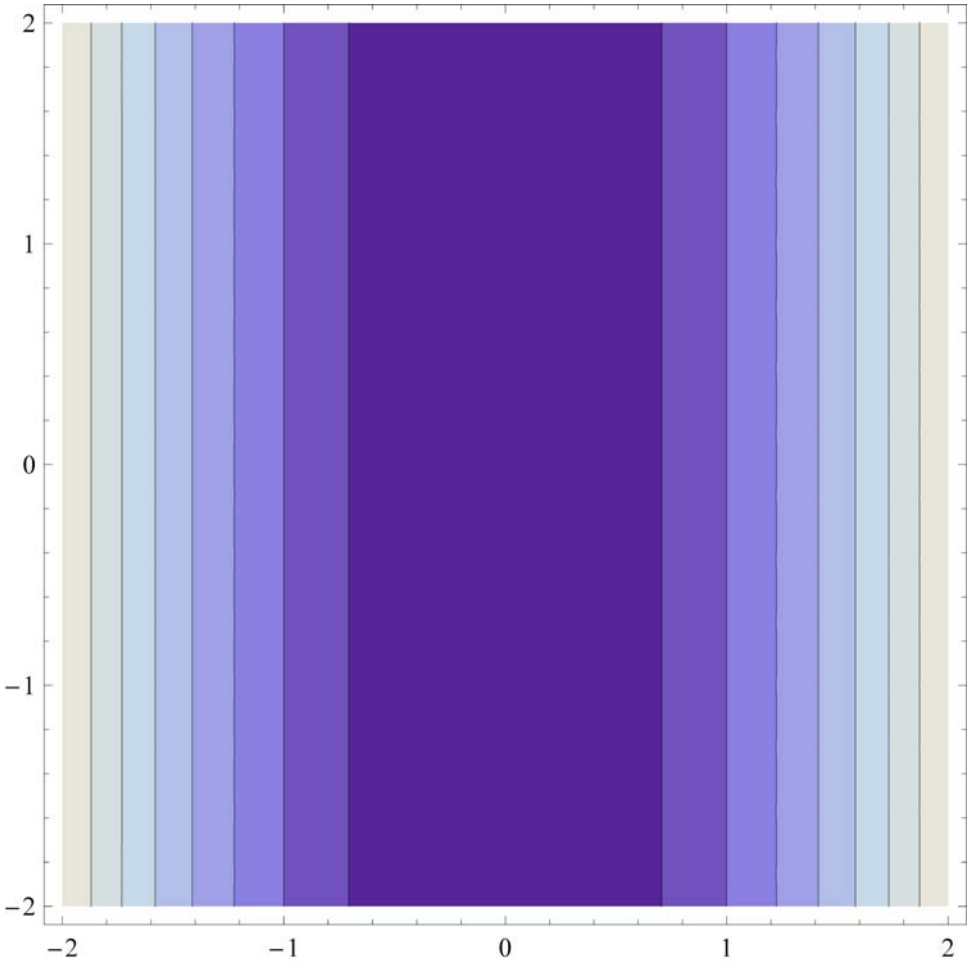
```
pl = x^2
```

x^2

```
gpl
```




```
vrstvy
```

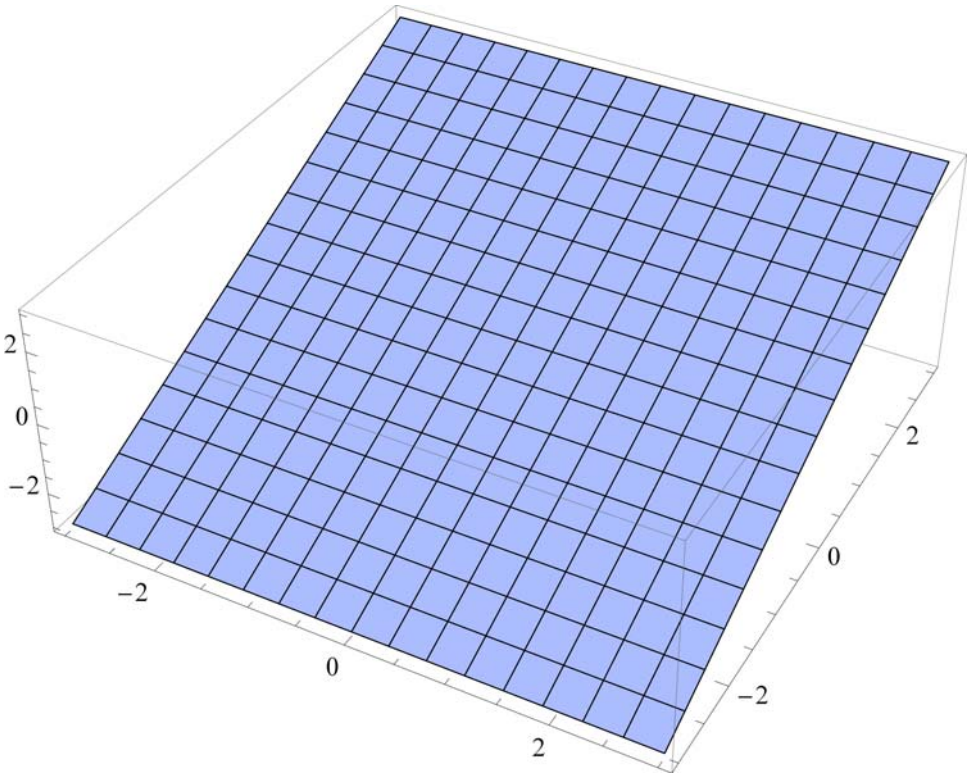


■ Nakreslite plochu $y^2 - z^2 = 0$

```
pl = y
```

y

```
g1 = gpl
```

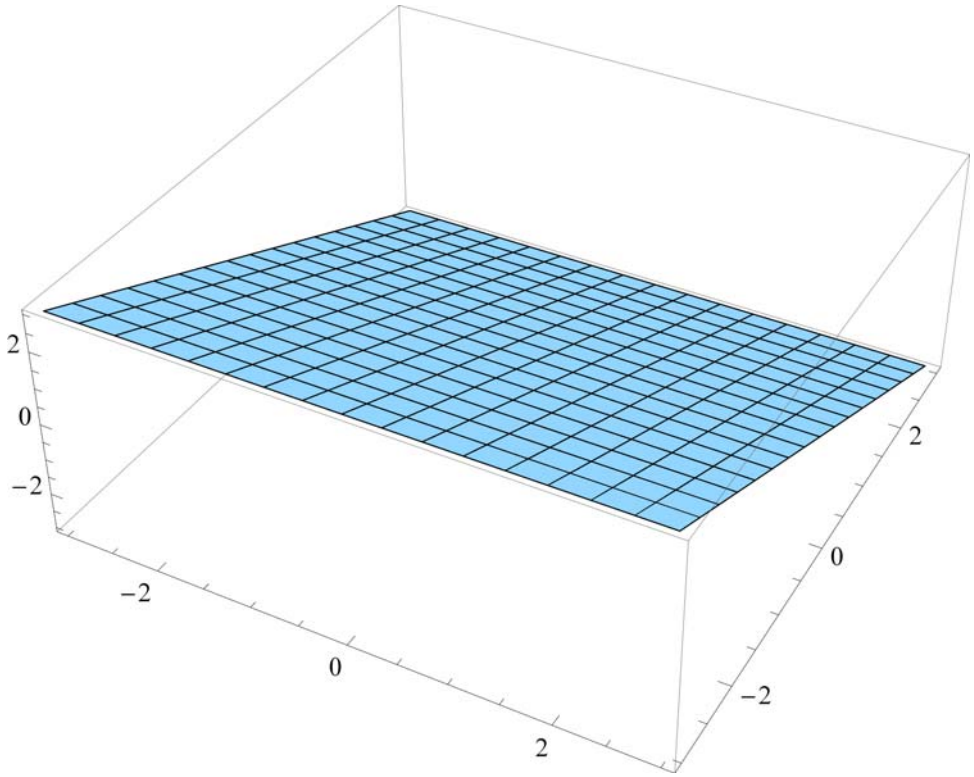



```
Clear[pl]
```

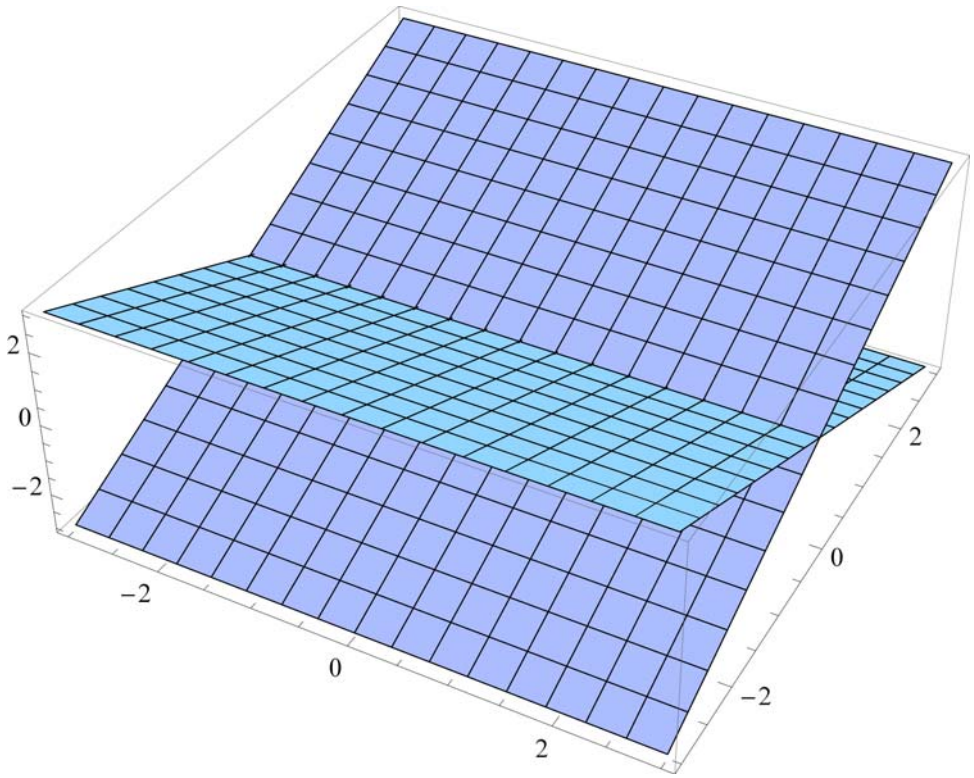
```
pl = -y
```

−y

```
g2 = gpl
```




```
Show[g1, g2]
```



```
Show[g1, g2, osx, osy, osz]
```