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ON SUBJECTED TO RAPID RANDOM EXTRACTIONS TWO-SEX POPULATION GROWTH

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Abstract: The paper analyzes a model for isolated population with sexual reproduction under assumption that the extractions are random and going at random time moments. Applying the approximative procedures of stochastic analysis we construct an ordinary differential equation for population dynamics in the mean and a linear stochastic differential equation for deviations on the mean trajectories. This approximative model permits to analyze a population growth as the Gaussian process with mean and variance given by ordinary differential equations.

Keywords: two-sex population; random extractions; stochastic approximation.

Mathematics Subject Classification: Primary 92B05,60H30; Secondary 60G57, 37H10.

1 Introduction

The dynamics of the two-sex populations has been discussed by many authors as the different forms of difference or differential equations (see, for example, [1 - 12] and references there). As has been mentioned by authors of the above papers and books the mathematical models may be used for qualitative and quantitative analysis of ecosystem, but also to manage the problems of agriculture, forest, animal husbandry and fishery ecosystem. The model construction is based on analysis of the increments $x(t_{k+1})-x(t_k)$ and $y(t_{k+1})-y(t_k)$ for male and female densities $\{x(t_k), k\in \mathbf{N}\}$ and $\{y(t_k), k\in \mathbf{N}\}$, where $\{0 < t_1 < t_2 <...< T\}$ are real numbers. Our paper analyze the popular in mathematical biology the *Liu* two-sex population model [9] with jump type random extractions at the below defined (1.1) time moments. We assume that the dynamics of population is given by the system of equations for the male and female densities $\{x_\varepsilon(t), y_\varepsilon(t), t\geq 0\}$. The corresponding to our model dynamical system may be described as follows:

- at time moment τ_k the phase coordinates have jumps to point

$$x_\varepsilon(\tau_k)=x_\varepsilon(\tau_k-)-\varepsilon h(\xi_k)x_\varepsilon(\tau_k-),\ y_\varepsilon(\tau_k)=y_\varepsilon(\tau_k-)-\varepsilon g(\xi_k)y_\varepsilon(\tau_k-)\,, \qquad (1.1)$$

where $x(\tau_1 -) := \lim_{t \uparrow \tau_1} x(t), y(\tau_1 -) = \lim_{t \uparrow \tau_1} y(t)$;

- further, at any time interval $(\tau_{k-1}, \tau_k), \mathrm{k} \in \mathbf{N}$ the population dynamics is given by differential equation

$$\frac{dx_\varepsilon(t)}{dt} = \alpha_1 y_\varepsilon(t) - \beta_1 x_\varepsilon(t) - K^{-1}(x_\varepsilon(t) + y_\varepsilon(t))x_\varepsilon(t), \frac{dy_\varepsilon(t)}{dt} = (\alpha_2 - \beta_2) y_\varepsilon(t) - K^{-1}(x_\varepsilon(t) + y_\varepsilon(t)) y_\varepsilon(t), \qquad (1.2)$$

where α_1 and α_2 are per capita birth rate for males and females, β_1 and β_2 are per capita death rate for males and females, K - is carrying capacity of population, $\{\xi_k, k \in \mathbf{N}\}$ are uniform R(0,1) distributed random variables, $\{h(\xi), g(\xi), \xi \in [0,1]\}$ are bounded nonnegative functions,

$$\begin{aligned} &\mathbf{E}\{h(\xi_k)\} \equiv \int_0^1 h(\xi) d\xi = \gamma_1, \mathbf{E}\{g(\xi_k)\} \equiv \int_0^1 g(\xi) d\xi = \gamma_2, \mathbf{E}\{h^2(\xi_k)\} \equiv \int_0^1 h^2(\xi) d\xi = \sigma_1^2, \\ &\mathbf{E}\{g^2(\xi_k)\} \equiv \int_0^1 g^2(\xi) d\xi = \sigma_2^2, \mathbf{E}\{h(\xi_k) g(\xi_k)\} \equiv \int_0^1 h(\xi) g(\xi) d\xi = \sigma_{12}^2 \end{aligned} \qquad (1.3)$$

and $\{\tau_k - \tau_{k-1}, k \in \mathbf{N}\}$ are independent on $\{\xi_k, k \in \mathbf{N}\}$ independent identically exponentially distributed with parameter ε^{-1}. In the next section applying stochastic averaging procedure [15] we proof that $\{x_\varepsilon(t), y_\varepsilon(t)\}$ are such continuously dependent on parameter ε functions , that there exists

$$p \lim_{\varepsilon \to 0} x_\varepsilon(t) = \bar{x}(t), p \lim_{\varepsilon \to 0} y_\varepsilon(t) = \bar{y}(t) \qquad (1.4)$$

and derive the ordinary differential equations for $\bar{x}(t), \bar{y}(t)$. In the third section we proof that for any *T>0* the distributions of the random processes $\left\{ \frac{x_\varepsilon(t) - \bar{x}(t)}{\sqrt{\varepsilon}}, t \in [0,T] \right\}$ and $\left\{ \frac{y_\varepsilon(t) - \bar{y}(t)}{\sqrt{\varepsilon}}, t \in [0,T] \right\}$ may be approximated by distributions of Gaussian processes, which satisfy the stochastic differential equations. The impulsive differential equation (1.1)-(1.2) defines two dimensional homogeneous Markov process. This process may be uniquely defined by the infinitesimal generator [13], which one can find as the limit

$$(L(\varepsilon)v)(x,y) := \lim_{t \downarrow 0} \frac{1}{t} \mathbf{E}\{v(x_\varepsilon(t), y_\varepsilon(t)) / x_\varepsilon(0) = x, y_\varepsilon(0) = y\} - v(x,y)$$

for an arbitrary sufficiently smooth bounded function $v(x,y)$. Using the asymptotic equalities

$$\mathbf{P}(\xi(t)\le x/\xi(0)=\eta,t<\Delta_1)=\mathbf{P}(\eta\le x)\mathbf{P}(t<\Delta_1)=\mathbf{P}(\eta\le x)e^{-\varepsilon^{-1}t}=\mathbf{P}(\eta\le x)+o(t),$$
$$\mathbf{P}(\xi(t)\le x/\xi(0)=\eta,t\ge\Delta_1)=\mathbf{P}(\xi_1\le x)\ 1-e^{-\varepsilon^{-1}t}\ +o(t)=\varepsilon^{-1}t\mathbf{P}(\xi_1\le x)+o(t)$$

we can find the above infinitesimal operator as follows:

$$(L(\varepsilon)v)(x,y)=v'_x(x,y)\left[\alpha_1 y-\beta_1 x-K^{-1}(x+y)x\right]+v'_y(x,y)\left[(\alpha_2-\beta_2)y-K^{-1}(x+y)y\right]+$$
$$+\varepsilon^{-1}\left[\int_0^1\mathbf{E}\{v(x-\varepsilon h(\xi)x,y-\varepsilon g(\xi)x)\}d\xi-v(x,y)\right] \tag{1.5}$$

2 The first approximation

As it has been proved in [14,15] if at any point $\{x,y\}\in\mathbf{R}^2$ for any sufficiently smooth function $v(x,y)$ there exists limit $\lim_{\varepsilon\to0}(L(\varepsilon)v)(x,y):=Lv(x,y)$ and L is the first order linear differential operator $L=a(x,y)\frac{\partial}{\partial x}+b(x,y)\frac{\partial}{\partial y}$ then we can approximate accurate within $o(\varepsilon)$ the expectations with the solutions of the smooth dynamical system with the vector flow $\{a(x,y),b(x,y)\}$. Corresponding to (1.5) limit operator for our Markov dynamical system (1.1)-(1.2) has a form

$$(Lv)(x,y)==\left[\alpha_1 y-(\beta_1+\gamma_1)x-K^{-1}(x+y)x\right]\frac{\partial}{\partial x}v(x,y)+\left[(\alpha_2-\beta_2-\gamma_2)y-K^{-1}(x+y)y\right]\frac{\partial}{\partial y}v(x,y) \tag{2.1}$$

Therefore, the above mentioned approximative dynamical system has a form:

$$\begin{cases}\dfrac{d\overline{x}(t)}{dt}=\alpha_1\overline{y}(t)-(\beta_1+\gamma_1)\overline{x}(t)-K^{-1}(\overline{x}(t)+\overline{y}(t))\overline{x}(t),\\ \dfrac{d\overline{y}(t)}{dt}=(\alpha_2-\beta_2-\gamma_2)\overline{y}(t)-K^{-1}(\overline{x}(t)+\overline{y}(t))\overline{y}(t)\end{cases} \tag{2.2}$$

If $\{\overline{x}(t),t\ge0\}$ and $\{\overline{y}(t),t\ge0\}$ are the solutions of the equation (2.2) with initial conditions $\overline{x}(0)=\mathbf{E}\{x_0(0)\},\overline{y}(0)=\mathbf{E}\{y_0(0)\}$ then $\lim_{\varepsilon\to0}\sup_{0\le t\le T}\ |\mathbf{E}\{x_\varepsilon(t)\}-\overline{x}(t)|+|\mathbf{E}\{y_\varepsilon(t)\}-\overline{y}(t)|\ =0$ for any T>0. The equation (2.2) has two equilibrium points: $A_1=\{0,0\}$ and $A_2=\{\overline{x},\overline{y}\}$, where

$$\overline{x}=\frac{K\alpha_1(\alpha_2-\beta_2-\gamma_2)}{\alpha_1+\beta_1+\gamma_1+\alpha_2-\beta_2-\gamma_2},\qquad \overline{y}=\frac{K(\alpha_2-\beta_2-\gamma_2)(\beta_1+\gamma_1+\alpha_2-\beta_2-\gamma_2)}{\alpha_1+\beta_1+\gamma_1+\alpha_2-\beta_2-\gamma_2} \tag{2.3}$$

The numbers (2.3) are positive if and only if the birth rate for females is sufficiently large: $\alpha_2>\beta_2+\gamma_2$. Not so difficult to ensure that under the above assumption the point $A_1=\{0,0\}$ is unstable knot and point A_2 is stable knot. The Fig. 1 contains two graphics: the solid lines are the solutions of equation (2.2) and dot lines are sample trajectories for solution of random impulsive differential equation (1.1) - (1.2) for parameters

$$\varepsilon = 0.01, K = 10,\ \alpha_1 = 0.7, \beta_1 = 0.4, \gamma_1 = 0.2, \alpha_2 = 1, \beta_2 = 0.4, \gamma_2 = 0.2\,. \tag{2.4}$$

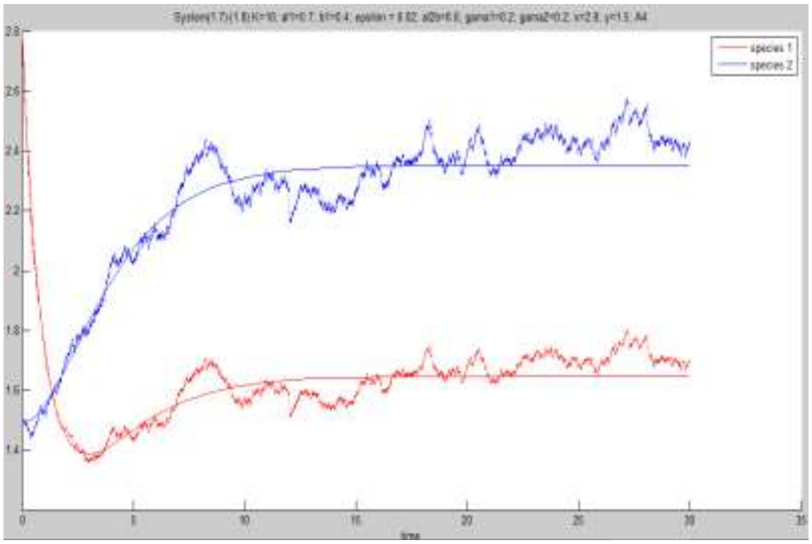

Fig. 1. The solution of equation (2.2) and the sample trajectories for solutions of the Markov dynamical system (1.1)-(1.2) for parameters (2.4).

As we can see there are sufficiently large deviations on population size averaged trajectory. To estimate these deviation, we need next step of the stochastic approximation procedure.

3 The normalized deviations on the first approximation

The second step for asymptotic analysis of the Markov impulsive differential equations (1.1)-(1.2) is diffusion approximation of the *normalized deviations* [14,15]:

$$z_\varepsilon(t) := \frac{x_\varepsilon(t) - \overline{x}(t)}{\sqrt{\varepsilon}}, u_\varepsilon(t) := \frac{y_\varepsilon(t) - \overline{y}(t)}{\sqrt{\varepsilon}} \tag{3.1}$$

The impulsive differential dynamical system for these processes has form ordinary differential equations:

$$\begin{aligned}
\frac{dz_\varepsilon(t)}{dt} &= \frac{1}{\sqrt{\varepsilon}}[\gamma_1 \overline{x}(t)] + \{\alpha_1 u_\varepsilon(t) - \beta_1 z_\varepsilon(t) - K^{-1}[(\overline{y}(t) + 2\overline{x}(t))z_\varepsilon(t) + u_\varepsilon(t)\overline{x}(t)]\} - \\
&\quad - \sqrt{\varepsilon}K^{-1}\{z_\varepsilon(t) + u_\varepsilon(t) z_\varepsilon(t)\} \\
\frac{du_\varepsilon(t)}{dt} &= \frac{1}{\sqrt{\varepsilon}}[\gamma_2 \overline{y}(t)] + \{(\alpha_2 - \beta_2)u_\varepsilon(t) - K^{-1}[(\overline{x}(t) + 2\overline{y}(t))u_\varepsilon(t) + z_\varepsilon(t)\overline{y}(t)] - \\
&\quad - \sqrt{\varepsilon}K^{-1}(z_\varepsilon(t) + u_\varepsilon(t))u_\varepsilon(t)
\end{aligned} \tag{3.2}$$

for $t \in (\tau_{k-1}, \tau_k), k \in \mathbf{N},$ and equations for jumps:

$$z_\varepsilon(\tau_k) = z_\varepsilon(\tau_k -) - \sqrt{\varepsilon} h(\xi_k)\bar{x}(\tau_k) - \varepsilon h(\xi_k) z_\varepsilon(\tau_k -),$$
$$u_\varepsilon(\tau_k) = u_\varepsilon(\tau_k -) - \sqrt{\varepsilon} g(\xi_k)\bar{y}(\tau_k) - \varepsilon g(\xi_k) u_\varepsilon(\tau_k -) \tag{3.3}$$

for $k \in \mathbf{N}$. The system of equations (2.2)-(3.2)-(3.3) defines on the probability space $(\Omega, \mathfrak{F}, \mathfrak{F}^t, \mathbf{P}, t \geq 0)$ four dimensional homogeneous Markov process $\{\bar{x}(t), \bar{y}(t), z_\varepsilon(t), u_\varepsilon(t), t \geq 0\}$. The weak infinitesimal operator for this process is defined by formula:

$$(\mathcal{L}(\varepsilon)v)(x,y,z,u) = \lim_{t \downarrow 0} \frac{1}{t}\Big[\mathbf{E}_{xy}^{zu}\{v(\bar{x}(t), \bar{y}(t), z_\varepsilon(t), u_\varepsilon(t))\} - v(x,y,z,u)\Big] =$$
$$= [\alpha_1 y - (\beta_1 + \gamma_1)x - K^{-1}(x+y)x]\frac{\partial}{\partial x}v(x,y,z,u) + [(\alpha_2 - \beta_2 - \gamma_2)y - K^{-1}(x+y)y]\frac{\partial}{\partial y}v(x,y,z,u) +$$
$$+\{\alpha_1 u - \beta_1 z - K^{-1}[(2x+y)z + ux]\}\frac{\partial}{\partial z}v(x,y,z,u) + \{(\alpha_2 - \beta_2)u - K^{-1}[(x+2y)u + zy]\}v\frac{\partial}{\partial u}(x,y,z,u) + \tag{3.4}$$
$$+\frac{1}{\sqrt{\varepsilon}}[\gamma_1 x]\frac{\partial}{\partial z}v(x,y,z,u) + \frac{1}{\sqrt{\varepsilon}}[\gamma_2 y]\frac{\partial}{\partial u}v(x,y,z,u) +$$
$$+\varepsilon^{-1}\int_0^1 v(x, y, z - \sqrt{\varepsilon}h(\xi)x - \varepsilon h(\xi)z, u - \sqrt{\varepsilon}g(\xi)y - \varepsilon g(\xi)u)\, d\xi$$

where $\mathbf{E}_{xy}^{zu}\{\bullet\} := \mathbf{E}\{\bullet / \bar{x}(0) = x, \bar{y}(0) = y, z_\varepsilon(0) = z, u_\varepsilon(0) = u\}$ and $v(x,y,z,u)$ sufficiently smooth bounded function. To apply stochastic approximation procedure [12] we have to calculate the limit in the formula (3.4) as $\varepsilon \to 0$. Under assumption that $v(x,y,z,u)$ is sufficiently smooth bounded function we can pass to a limit as $\varepsilon \to 0$:

$$(\hat{\mathcal{L}}v)(x,y,z,u) := \lim_{\varepsilon \to 0}(\mathcal{L}(\varepsilon)v)(x,y,z,u) = [\alpha_1 y - (\beta_1 + \gamma_1)x - K^{-1}(x+y)x - \gamma_1 z]\frac{\partial}{\partial x}v(x,y,z,u) +$$
$$+[(\alpha_2 - \beta_2 - \gamma_2)y - K^{-1}(x+y)y - \gamma_2 u]\frac{\partial}{\partial y}v(x,y,z,u) + xy\sigma_{12}^2\frac{\partial^2}{\partial z \partial u}v(x,y,z,u) +$$
$$+\{\alpha_1 u - \beta_1 z - K^{-1}[(2x+y)z + ux]\}\frac{\partial}{\partial z}v(x,y,z,u) + \frac{1}{2}\sigma_1^2 x^2\frac{\partial^2}{\partial z^2}v(x,y,z,u) + \tag{3.5}$$
$$+\{(\alpha_2 - \beta_2)u - K^{-1}[(x+2y)u + zy]\}\frac{\partial}{\partial u}v(x,y,z,u) + \frac{1}{2}\sigma_2^2 y^2\frac{\partial^2}{\partial u^2}v(x,y,z,u)$$

The operator $\hat{\mathcal{L}}$ may be interpret [13] as the weak infinitesimal operator of the diffusion Markov process $\{z(t), u(t), t \geq 0\}$ given by the linear nonhomogeneous Ito stochastic differential equations:

$$
\begin{cases}
dz(t) = [-K^{-1}(2\bar{x}(t) + \bar{y}(t)) - \beta_1]z(t)dt + [\alpha_1 - K^{-1}\bar{x}(t)]u(t)dt + \\
\qquad + \bar{\sigma}_1(t)dw_1(t) + \bar{\sigma}_{12}(t)dw_2(t), \\
du(t) = -K^{-1}\bar{y}(t)z(t)dt + [(\alpha_2 - \beta_2) - K^{-1}(\bar{x}(t) + 2\bar{y}(t))]u(t)dt + \\
\qquad + \bar{\sigma}_2(t)dw_1(t) + \bar{\sigma}_{12}(t)dw_2(t)
\end{cases}
\tag{3.6}
$$

with initial condition $z(0) = 0, u(0) = 0$, where $\{\bar{x}(t), \bar{y}(t), t \geq 0\}$ are solution of the equation (2.2) with initial conditions $\bar{x}(0) = x_\varepsilon(0), \bar{y}(0) = y_\varepsilon(0)$, $w_1(t), w_2(t)$ are the standard independent Wiener processes, and coefficients $\bar{\sigma}_1(t), \bar{\sigma}_2(t), \bar{\sigma}_{12}(t)$ are the nonnegative solutions of the matrix equations

$$
\begin{pmatrix} \bar{\sigma}_1(t) & \bar{\sigma}_{12}(t) \\ \bar{\sigma}_{12}(t) & \bar{\sigma}_2(t) \end{pmatrix}^2 = \begin{pmatrix} \sigma_1^2 \bar{x}^2(t) & \sigma_{12}^2 \bar{x}(t)\bar{y}(t) \\ \sigma_{12}^2 \bar{x}(t)\bar{y}(t) & \sigma_2^2 \bar{y}^2(t) \end{pmatrix}.
$$

The covariance matrix $Q(t) = \begin{pmatrix} q_{11}(t) & q_{12}(t) \\ q_{12}(t) & q_{22}(t) \end{pmatrix}$ for the solution of equation (3.6) with zero initial condition satisfies the differential equation

$$
\frac{d}{dt}Q(t) = AQ(t) + Q(t)A^T + \Sigma^2 \tag{3.7}
$$

with initial condition $Q(0) = 0$. Not so difficult to ensure that under assumption $\alpha_2 > \beta_2 + \gamma_2$ there exists $\lim_{t\to\infty} Q(t) = \hat{Q}$, where matrix $\hat{Q} = \begin{pmatrix} \hat{q}_{11} & \hat{q}_{12} \\ \hat{q}_{12} & \hat{q}_{22} \end{pmatrix}$ satisfies an algebraic equation

$$
A\hat{Q} + \hat{Q}A^T = -\Sigma^2 .
$$

As it has been proved in [14,15] on any finite dimensional interval [0,T] the finite dimensional distributions of the processes $\{x_\varepsilon(t), y_\varepsilon(t), 0 \leq t \leq T\}$ may be approximated by the Gaussian distributions of the processes $\{\bar{x}(t) + z(t)\sqrt{\varepsilon}, \bar{y}(t) + u(t)\sqrt{\varepsilon}, 0 \leq t \leq T\}$. As it has been mentioned the solution $\{\bar{x}(t), \bar{y}(t)\}$ of the average equation converges to the equilibrium point $\{\bar{x}, \bar{y}\}$ given by equation (2.3). Therefore, with the course of time the population concentrates in a neighborhood of equilibrium in the mean (2.3) and the equations (3.6) have more simple form:

$$
d\vec{z}(t) = A\vec{z}(t)dt + \Sigma d\vec{w}(t) , \tag{3.8}
$$

where

$$\vec{z}(t)=\begin{pmatrix} z(t) \\ u(t) \end{pmatrix}, \vec{w}(t)=\begin{pmatrix} w_1(t) \\ w_2(t) \end{pmatrix}, \Sigma=\begin{pmatrix} \bar{\sigma}_1 & \bar{\sigma}_{12} \\ \bar{\sigma}_{12} & \bar{\sigma}_2 \end{pmatrix},$$

$$A=\begin{pmatrix} -K^{-1}(2\bar{x}+\bar{y})-\beta_1 & \alpha_1-K^{-1}\bar{x} \\ -K^{-1}\bar{y} & \alpha_2-\beta_2-K^{-1}(\bar{x}+2\bar{y}) \end{pmatrix}$$

and coefficients $\bar{\sigma}_1,\bar{\sigma}_2,\bar{\sigma}_{12}$ are defined as the positive solution of the algebraic equation:

$$\bar{\sigma}_1^2+\bar{\sigma}_{12}^2=\sigma_1^2\bar{x}^2,\ \ \bar{\sigma}_1+\bar{\sigma}_2\ \ \bar{\sigma}_{12}=\sigma_{12}^2\overline{xy}, \bar{\sigma}_2^2+\bar{\sigma}_{12}^2=\sigma_2^2\bar{y}^2 .$$

The solution of the equation (3.8) with initial condition $\vec{z}(t_0)=\vec{z}_0$ is the two dimensional Gaussian vector process given by formula:

$$\vec{z}(t,t_0,z_0)=\exp\ (t-t_0)A\ \ \vec{z}_0+\int_{t_0}^{t}\exp\ (t-s)A\ \ \Sigma d\vec{w}(s)\ . \tag{3.9}$$

Using the solutions (3.9) with initial values $t_0=0, \vec{z}_0=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ we can approximate the solution $\vec{X}_\varepsilon(t)=\begin{pmatrix} x_\varepsilon(t) \\ y_\varepsilon(t) \end{pmatrix}$ of the impulsive differential equation (1.1) - (1.2) as follows:

$$\vec{X}_\varepsilon(t)\approx\vec{\bar{\bar{X}}}(t)+\sqrt{\varepsilon}\int_0^t\exp\ (t-s)A\ \ \Sigma d\vec{w}(s), \tag{3.10}$$

where $\vec{\bar{\bar{X}}}(t)$ is the vector-solution of the equation (2.2) with initial condition $\vec{\bar{\bar{X}}}(0)=\vec{X}_\varepsilon(0)$.

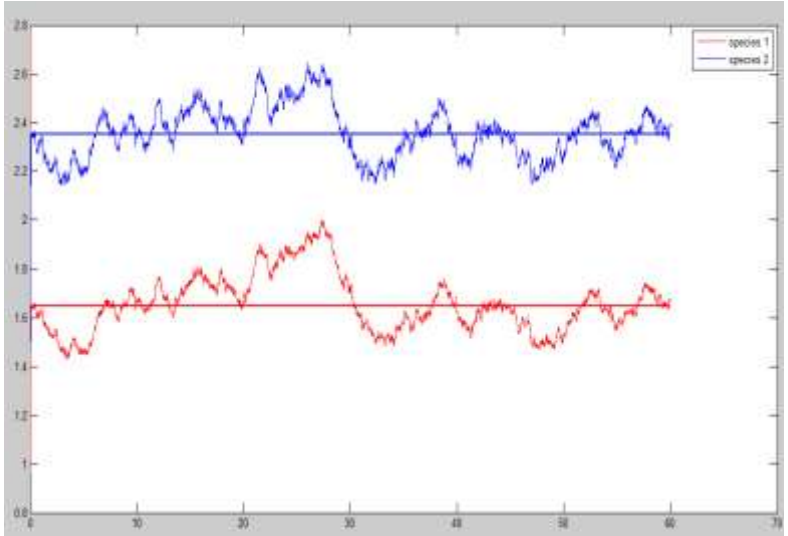

Fig. 2. The solution of equation (2.2) with initial condition (2.3)
and the sample trajectories for the Gaussian approximation.

As has been mentioned in the second section, if $\alpha_2 > \beta_2 + \gamma_2$ then the eigenvalues of the matrix A are negative. Therefore the stochastic integral in (3.9) converges as $t_0 \to -\infty$ with probability one and [14] there exists satisfying to (3.8) stationary stochastic process

$$\lim_{t_0 \to -\infty} \vec{z}(t, t_0, z_0) = \vec{Z}_0(t) = \begin{pmatrix} \hat{z}(t) \\ \hat{u}(t) \end{pmatrix} = \int_{-\infty}^{t} \exp\ (t-s)A\ \Sigma d\vec{w}(s) \qquad (3.11)$$

with zero mean and the symmetric constant covariance matrix $\hat{Q}$.

4 Conclusions

Remember that by definition the process $\vec{Z}_0(t)$ is the solution of the stochastic differential equation (3.8) with initial conditions $\vec{z}_0(0) = \int_{-\infty}^{0} \exp\ -sA\ \Sigma d\vec{w}(s)$. Therefore $\vec{V}(t) = \vec{Z}_0(t) - \vec{z}(t, t_0, z_0)$ is deterministic vector-function that satisfies a homogeneous ordinary differential equation in $\mathbf{R}^2$ and may be given as the matrix exponent $\vec{V}(t) = \hat{Z}(0)\exp\{At\}$, where A is a matrix with negative eigenvalues. This means that under assumption $\alpha_2 > \beta_2 + \gamma_2$ the population size stabilizes at the $\sqrt{\varepsilon}$ – neighbourhood of the defined by formula (2.3) equilibrium point $\vec{x} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$ and for sufficiently large $t > 0$ the random variable $\{x_\varepsilon(t), y_\varepsilon(t)\}$ may be analyzed as two dimensional normal distributed random variable with mean $\vec{x}$ and covariance matrix $\varepsilon\hat{Q}$.

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